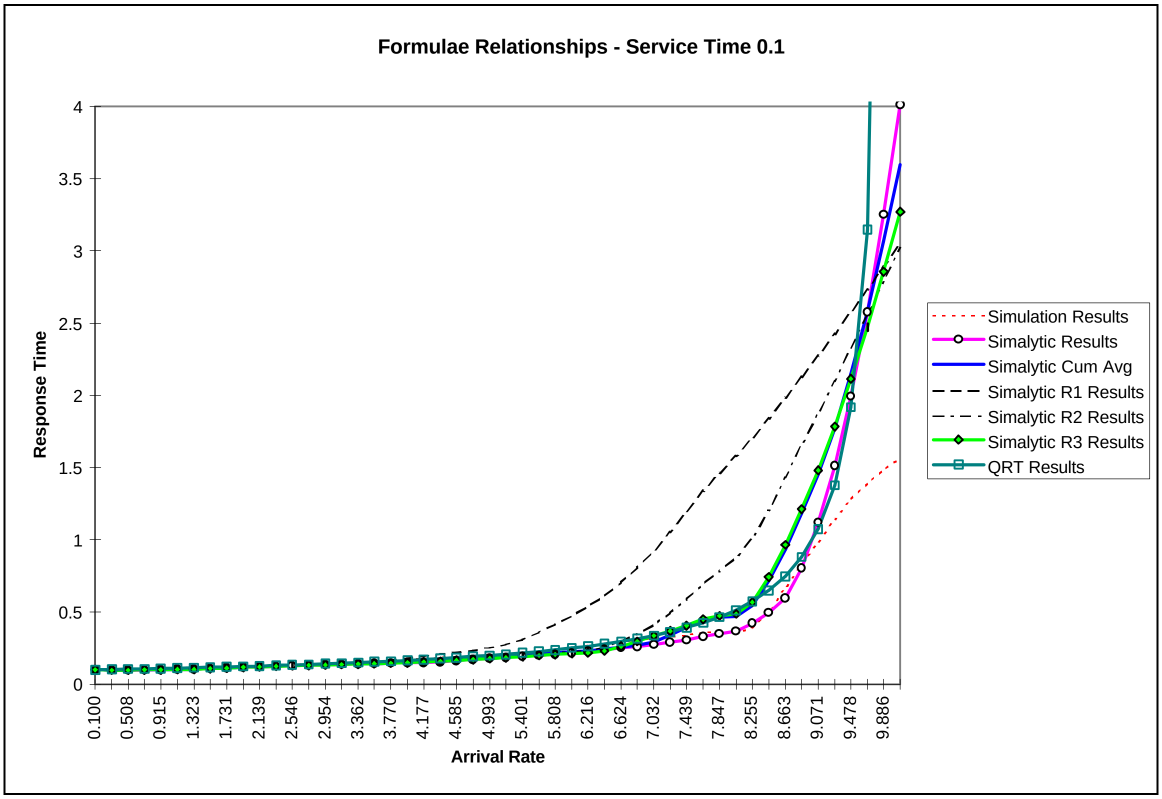
7.5 Appendix E: MathCAD Formulae Results Charts

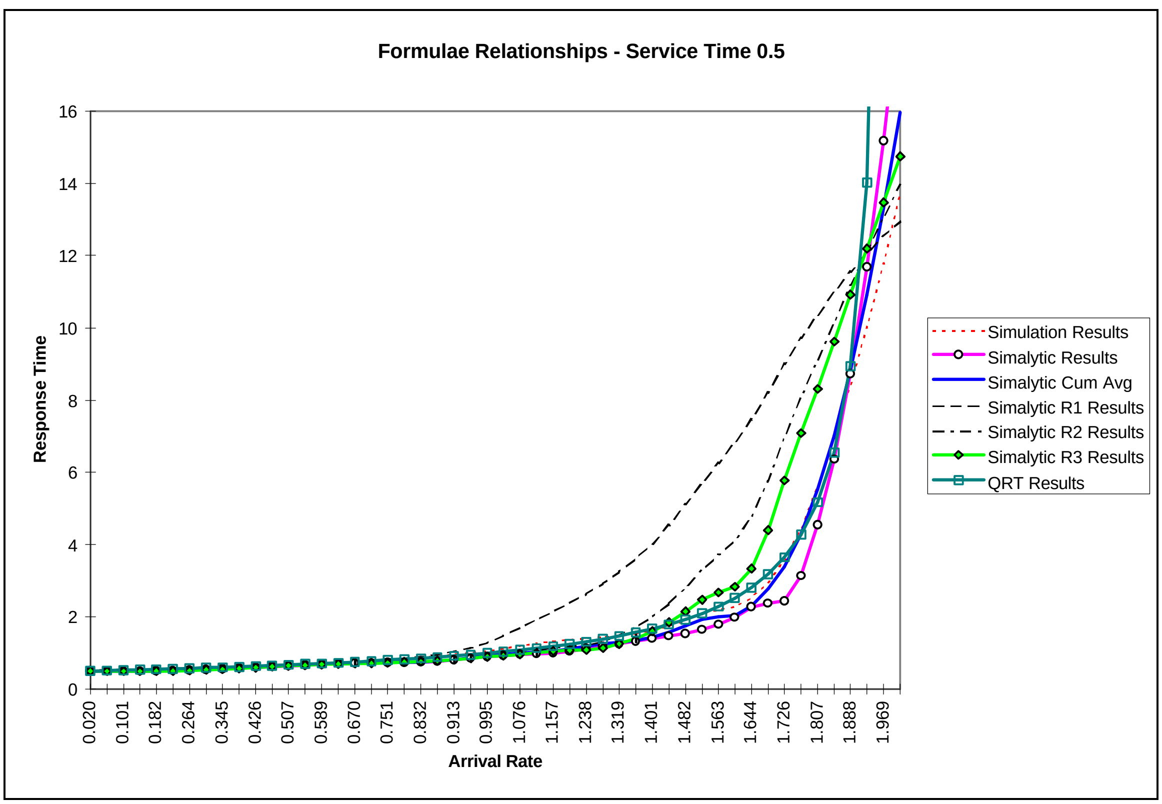
The following Microsoft Excel charts show the detailed results of the mathematical formulae as implemented using MathCAD. The MathCAD worksheet used to generate the results is listed in *Figure 60 MathCAD Worksheet Listing* on page 199. The data was collected into a spreadsheet from MathCAD output files. Each set of data is copied to a worksheet and then charted.

7.5.1 Formulae Relationships

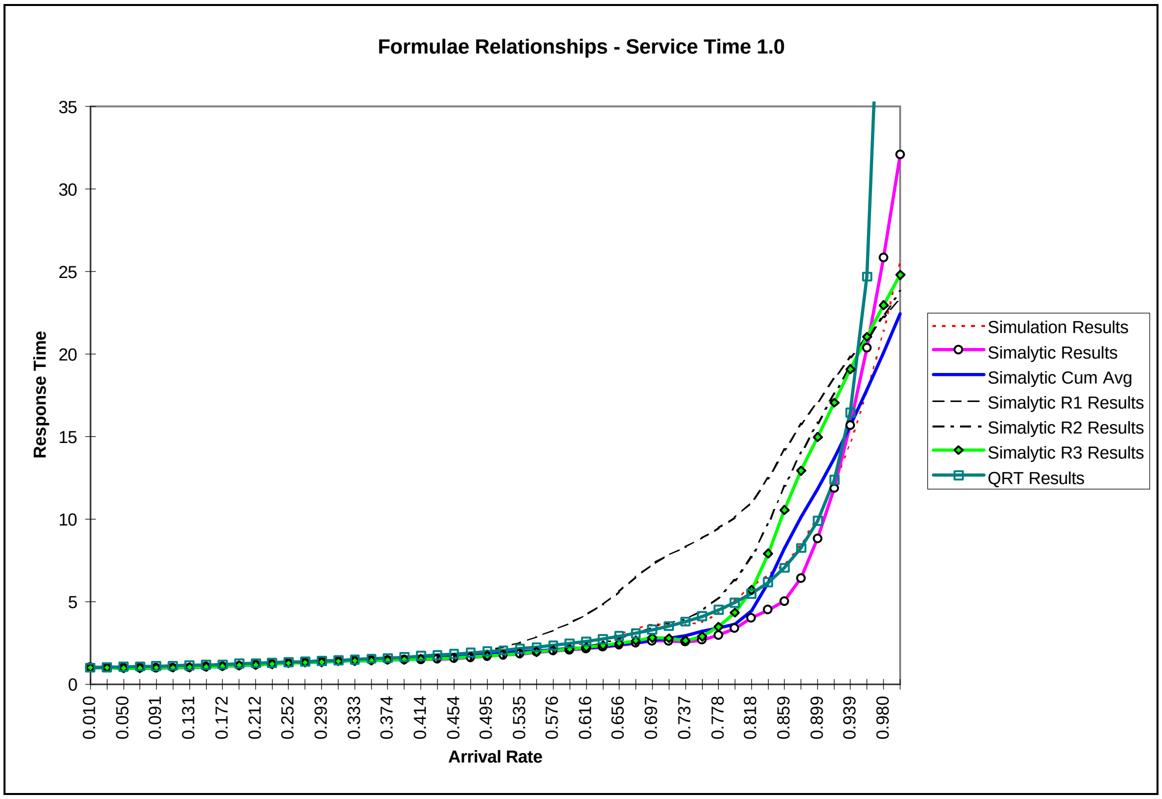
The first group of charts, labeled **Formulae Relationships - Service Time**, shows the relationships between the three formulae (*Equation 1 Analytic Response Time Formula* on page 32, *Equation 2 Simulation Response Time Formula* on page 35 and *Equation 3 Simalytic Response Time Formula* on page 61) implemented in MathCAD at a series of arrival rates as described in section *3.5.2 Verification/Validation Approach* on page 81. The line labeled **Simulation Results** shows the response times calculated by a single execution of the simplistic simulation technique in MathCAD. Some anomalies are evident for this line in some of the charts because of the effect the arrival distribution variation has on a single model run. These anomalies, seen as curve fall-off at high arrival rates, disappear when the model is rerun using a different random number seed. The line labeled **QRT Results** shows the response times calculated by the MathCAD formula (QueuingRT) for queuing theory response time in *Figure 59 MathCAD Worksheet for Queuing Theory Surface Plots* on page 178. The lines labeled **Simalytic** show results calculated using different implementations of the Simalytic function. **Simalytic Results** shows the basic values in the Simalytic function response time table where the function is called once for each arrival rate with the average of all interarrvial times. This is the best value that can be returned if the arrival rate is consistent across the sample period (i.e. a steady-state system). **Simalytic Cum Avg** shows the results when the function calculates the cumulative average of the interarrival times each time it is called and returns the response time from the table for that average. **Simalytic R1 Results, Simalytic R2 Results** and **Simalytic R3 Results** show the results when the function calculates a rolling average of the interarrival times each time it is called. R1 is a small rolling window (1.5% of the total events), R2 is a medium rolling window (5% of the total events) and R3 is a large rolling window (10% of the total events). In each case, the function sums the last *x* events (where *x* represents the rolling window size), divides by *x* and returns the response time from the table for that average. Each of these implementations is a compromise between sensitivity to short-term dynamics and consistent long-term results. The R1 results provide the greatest sensitivity to dynamic changes but experimentation has shown that the R1 results, and to a lessor degree the R2 results, are overly sensitive to distribution event order and look-up table step values. The R3 results were used in the comparisons to the simulation and queuing theory results because they provide the closest approximation to the steady-state system results but are calculated from parameters to the function and still remain sensitive to dynamic changes in the arrival rate patterns.

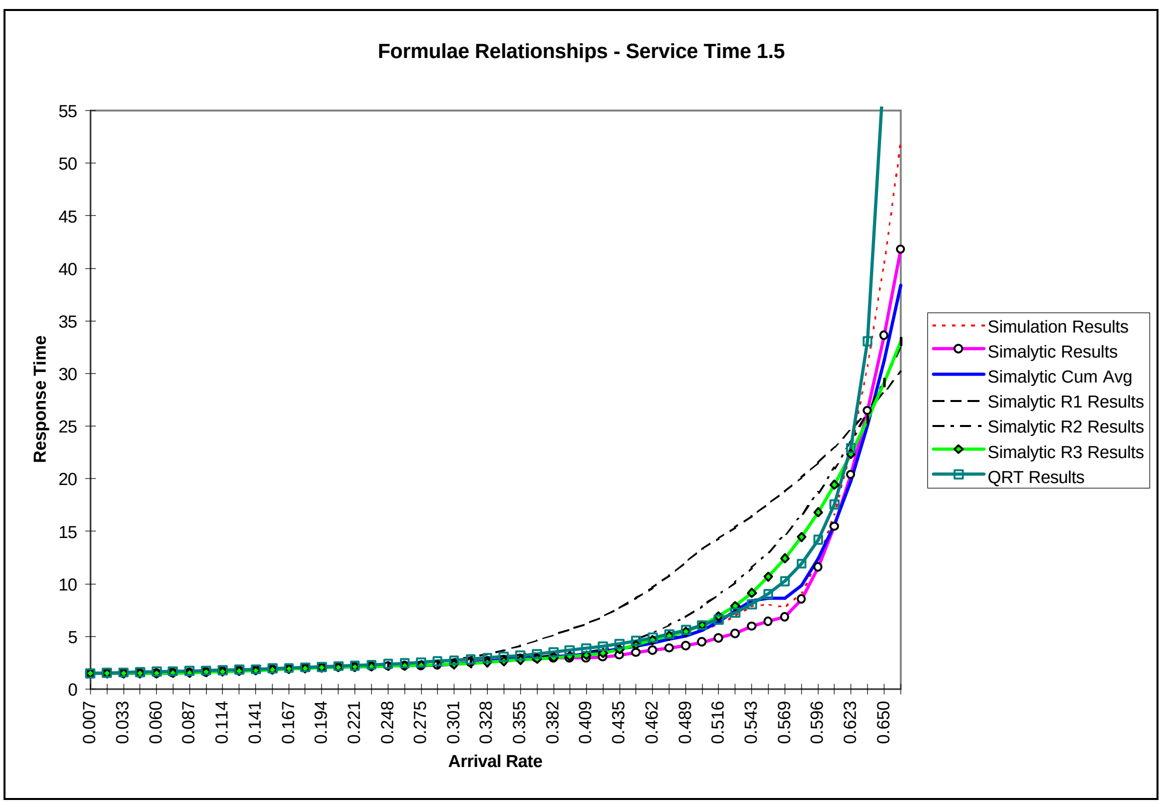


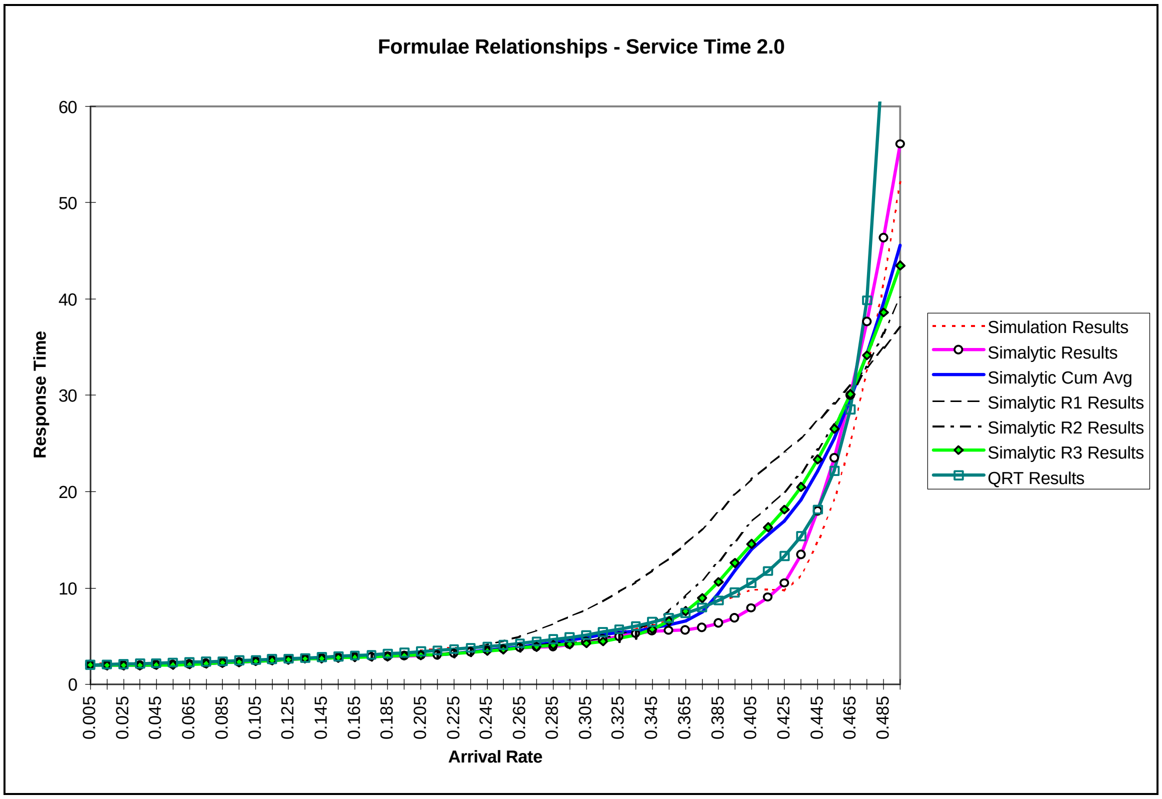
Formulae Relationships - Service Time 0.1
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

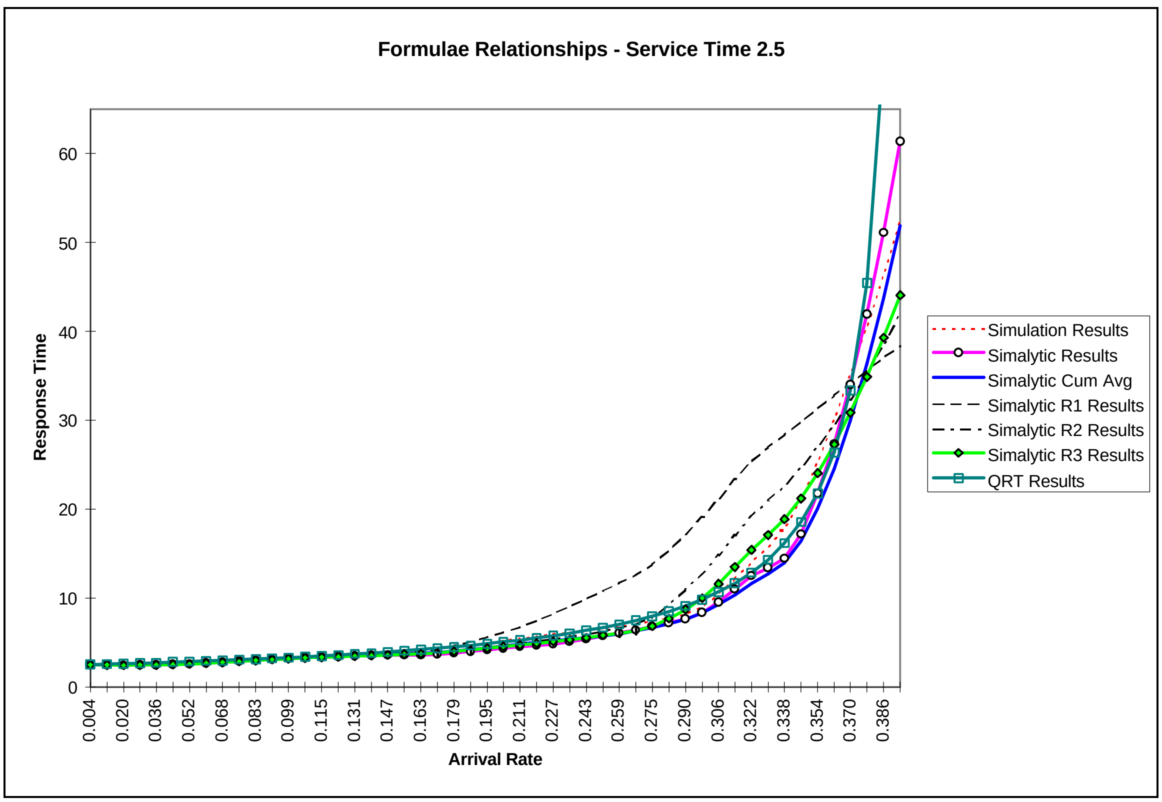


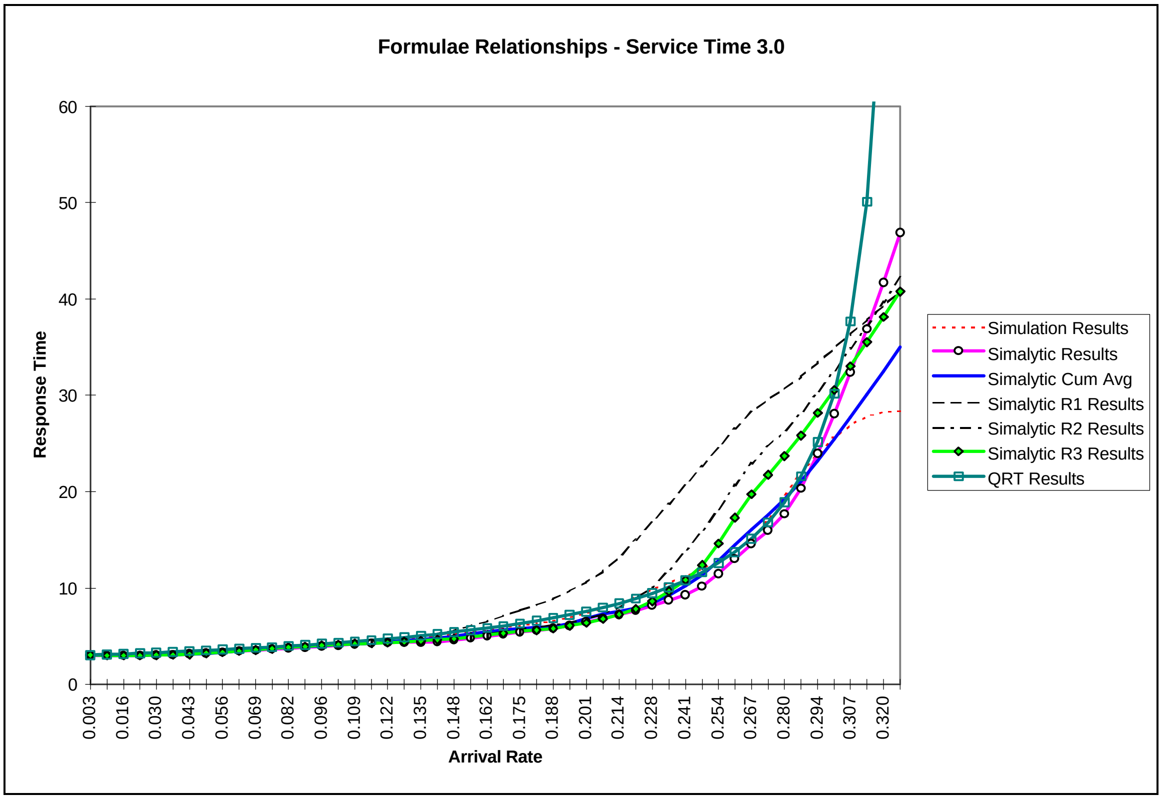
Formulae Relationships - Service Time 0.5
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

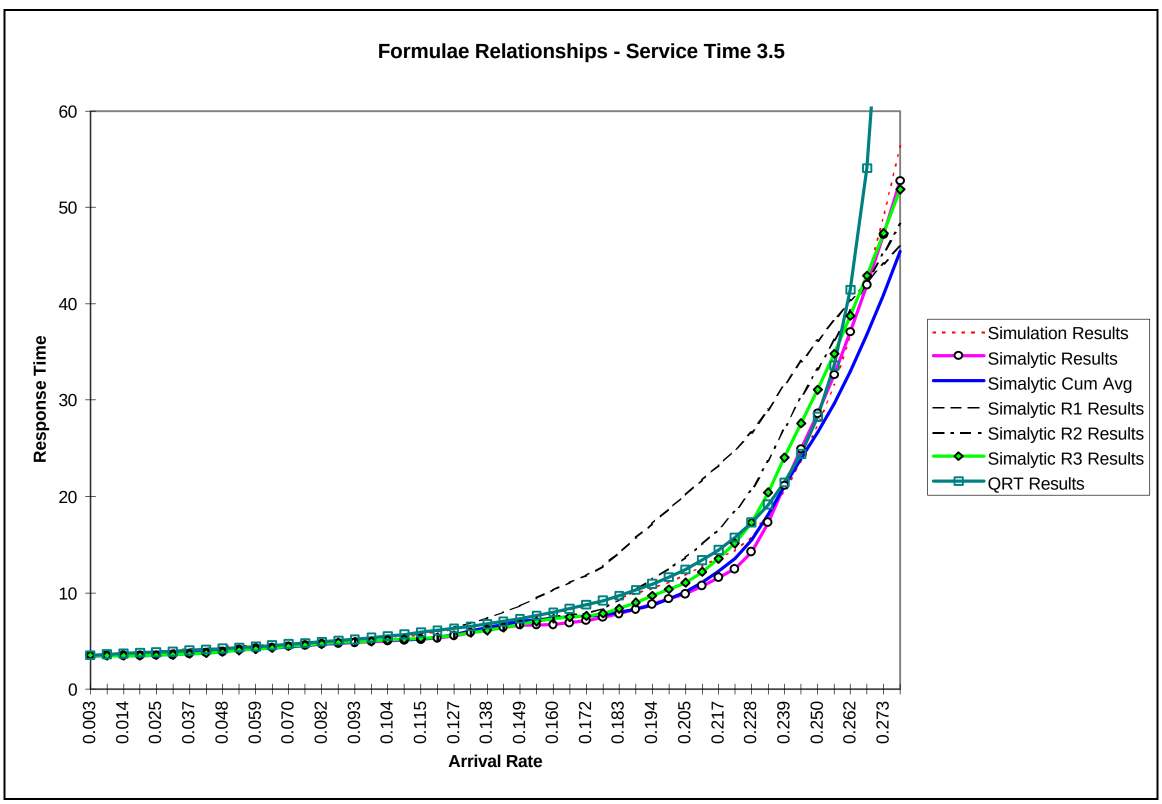

Formulae Relationships - Service Time 1.0
Response Time
35
30
25
20
15
10
5
0
0.010
0.050
0.091
0.131
0.172
0.212
0.252
0.293
0.333
0.374
0.414
0.454
0.495
0.535
0.576
0.616
0.656
0.697
0.737
0.778
0.818
0.859
0.899
0.939
0.980
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

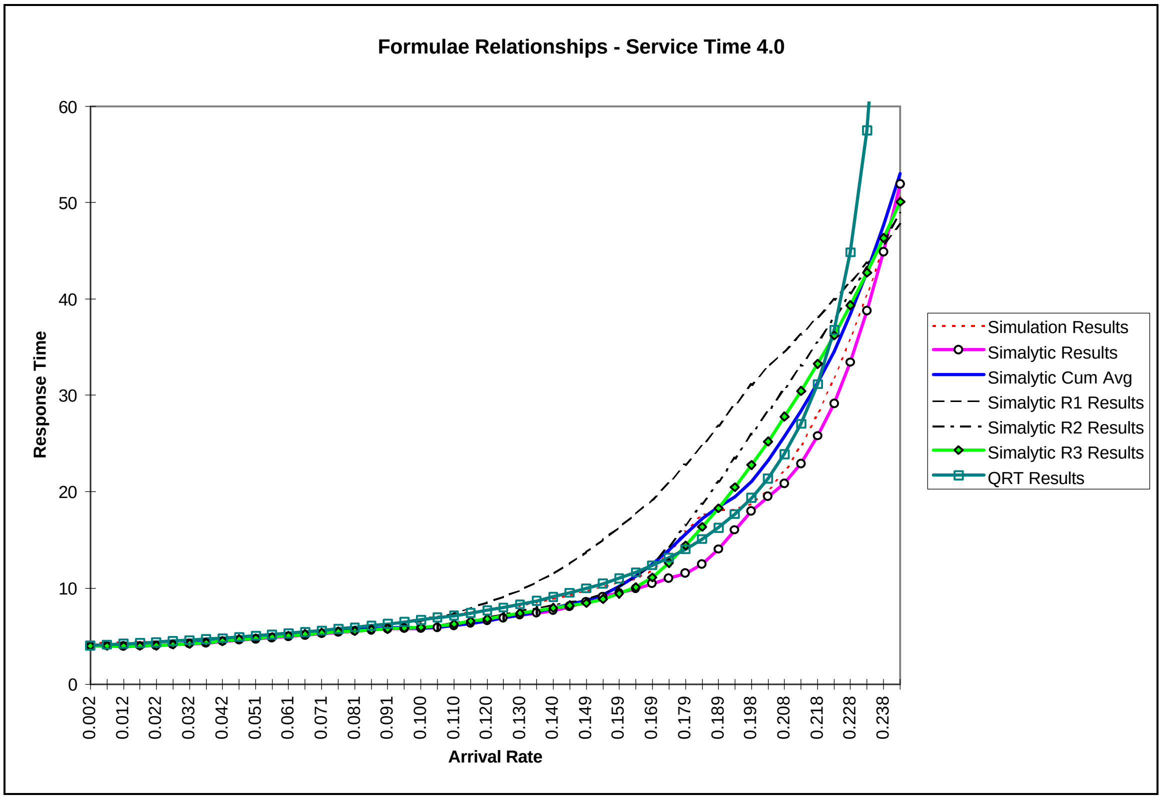

Formulae Relationships - Service Time 1.5
Response Time
55
50
45
40
35
30
25
20
15
10
5
0
0.007
0.033
0.060
0.087
0.114
0.141
0.167
0.194
0.221
0.248
0.275
0.301
0.328
0.355
0.382
0.409
0.435
0.462
0.489
0.516
0.543
0.569
0.596
0.623
0.650
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

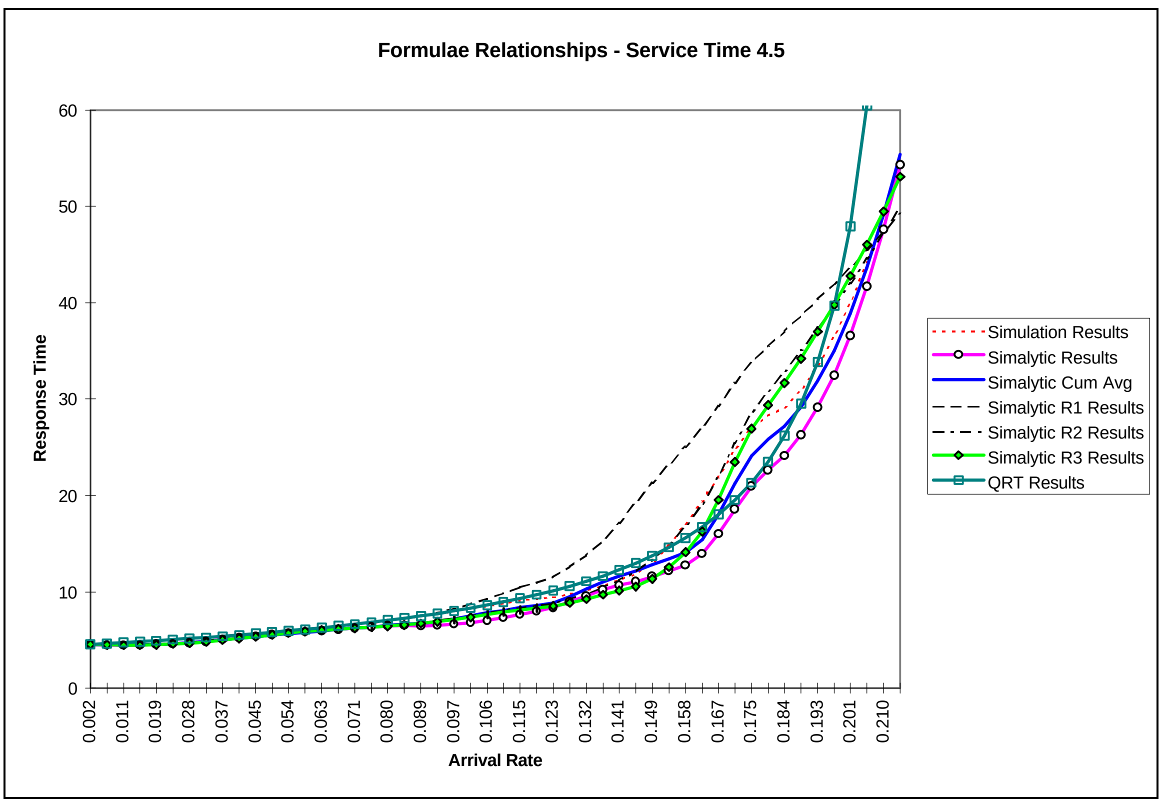

Formulae Relationships - Service Time 2.0
Response Time
0
10
20
30
40
50
60
Arrival Rate
0.005 0.025 0.045 0.065 0.085 0.105 0.125 0.145 0.165 0.185 0.205 0.225 0.245 0.265 0.285 0.305 0.325 0.345 0.365 0.385 0.405 0.425 0.445 0.465 0.485
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

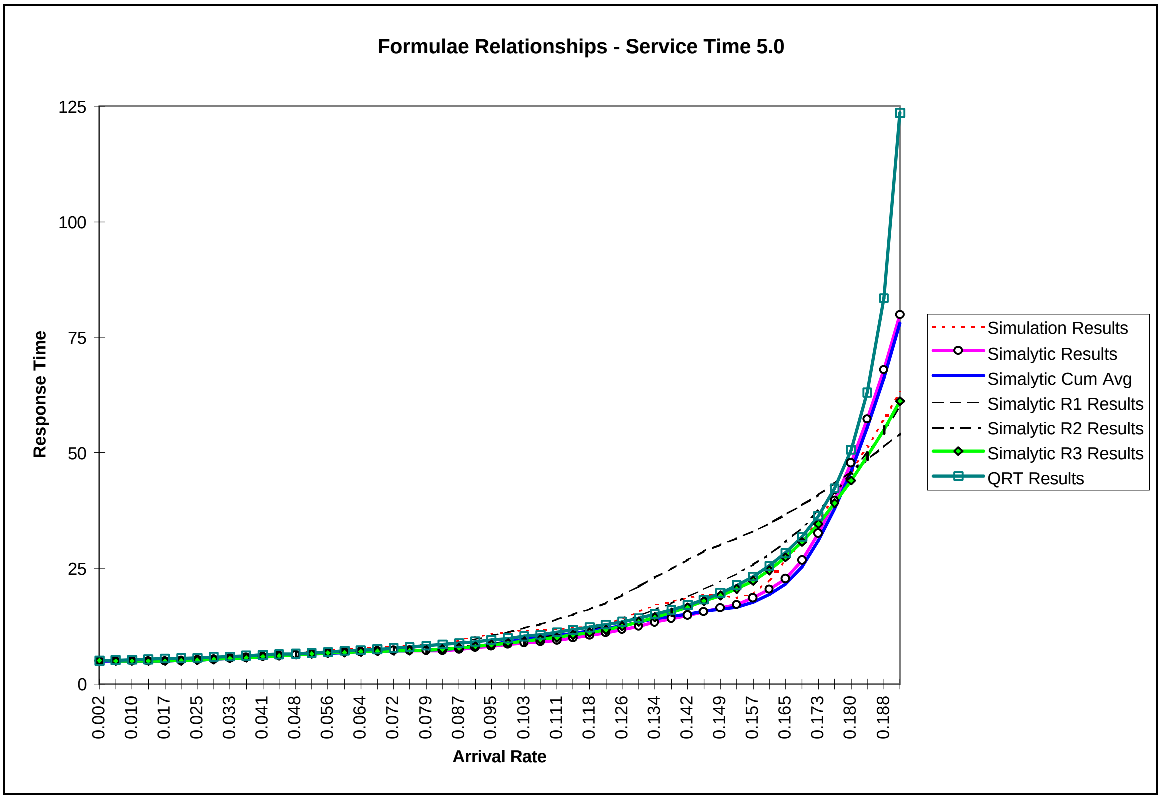

Formulae Relationships - Service Time 2.5
Response Time
0
10
20
30
40
50
60
Arrival Rate
0.004 0.020 0.036 0.052 0.068 0.083 0.099 0.115 0.131 0.147 0.163 0.179 0.195 0.211 0.227 0.243 0.259 0.275 0.290 0.306 0.322 0.338 0.354 0.370 0.386
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

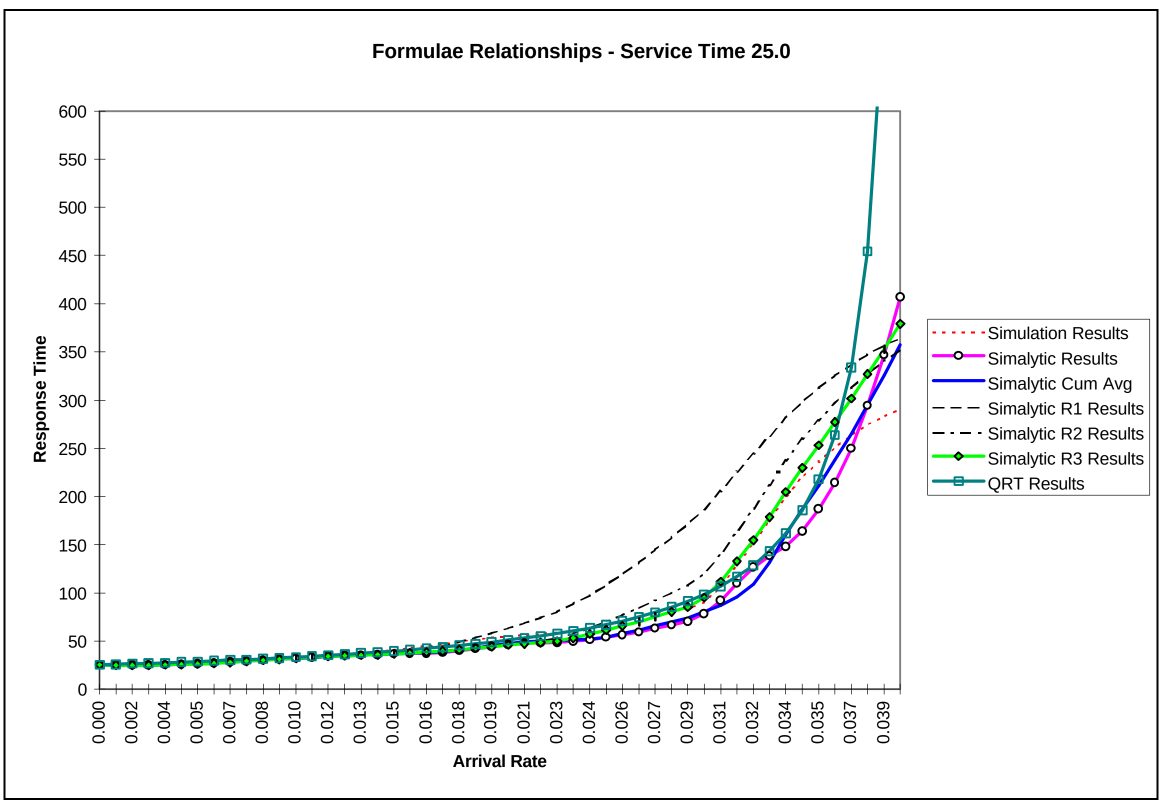

Formulae Relationships - Service Time 3.0
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

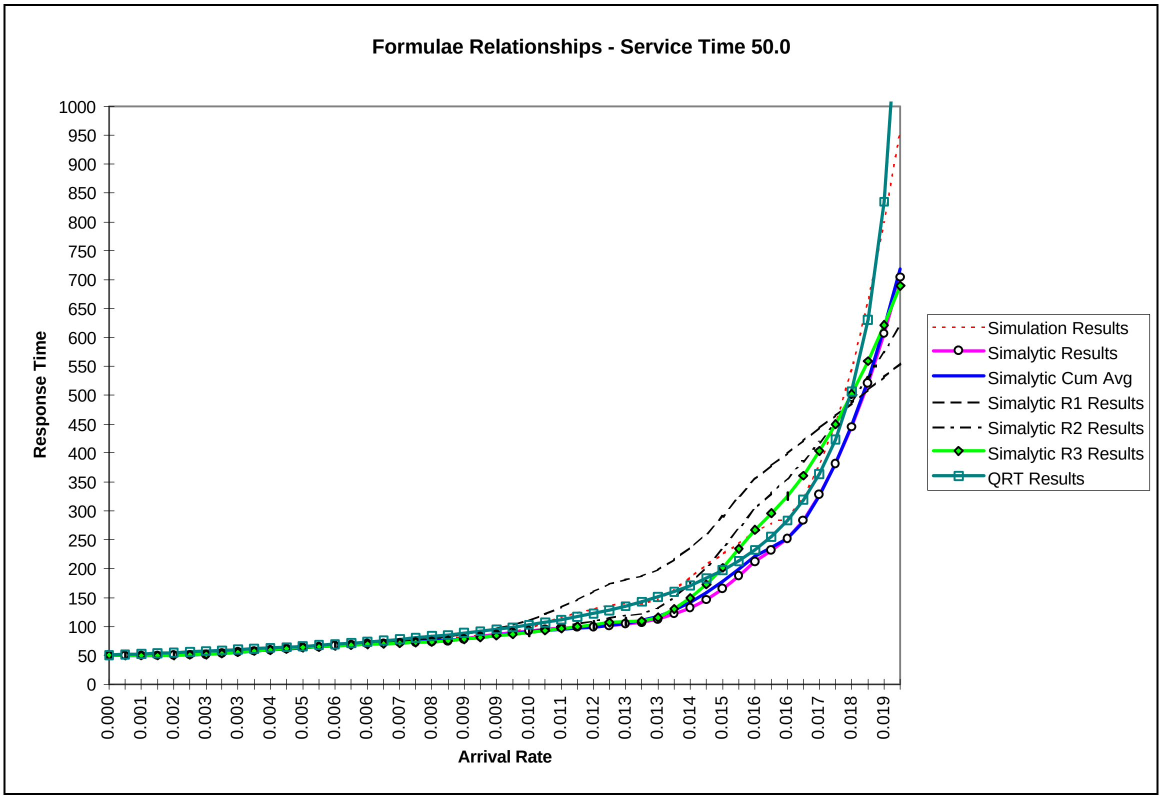

Formulae Relationships - Service Time 3.5
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

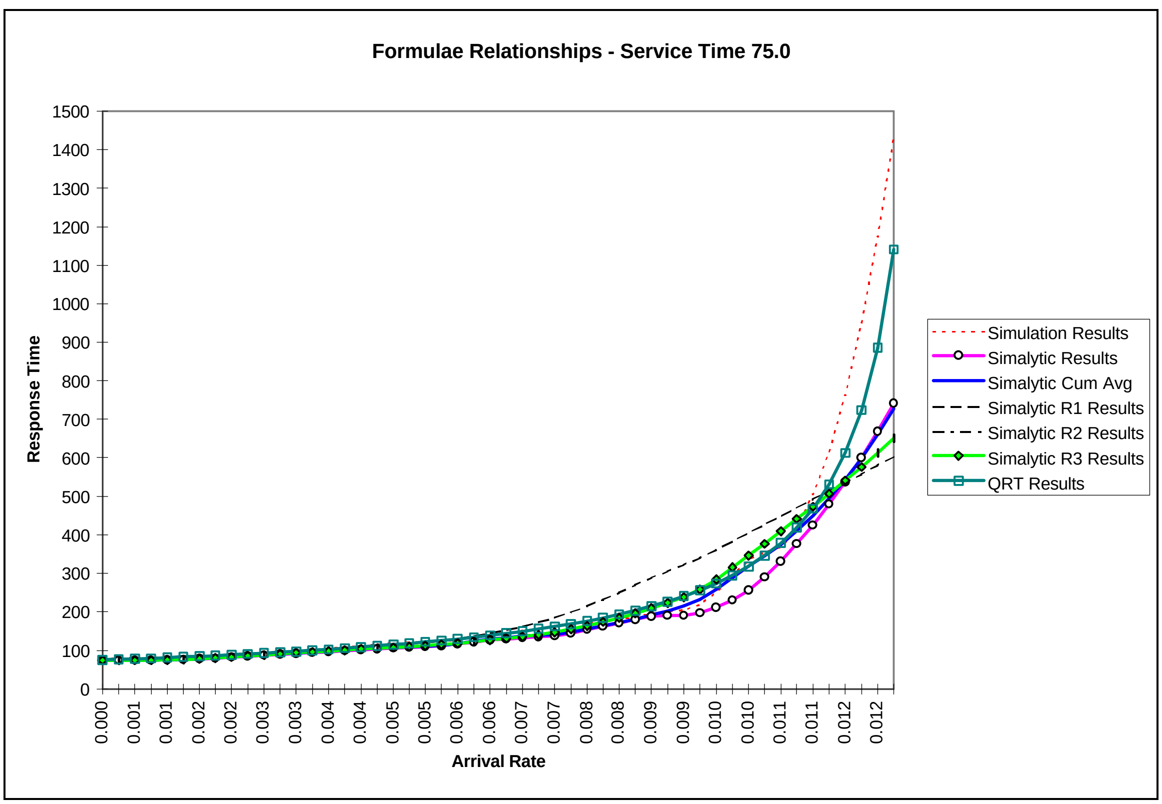

Formulae Relationships - Service Time 4.0
Response Time
Arrival Rate
0
10
20
30
40
50
60
0.002
0.012
0.022
0.032
0.042
0.051
0.061
0.071
0.081
0.091
0.100
0.110
0.120
0.130
0.140
0.149
0.159
0.169
0.179
0.189
0.198
0.208
0.218
0.228
0.238
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results

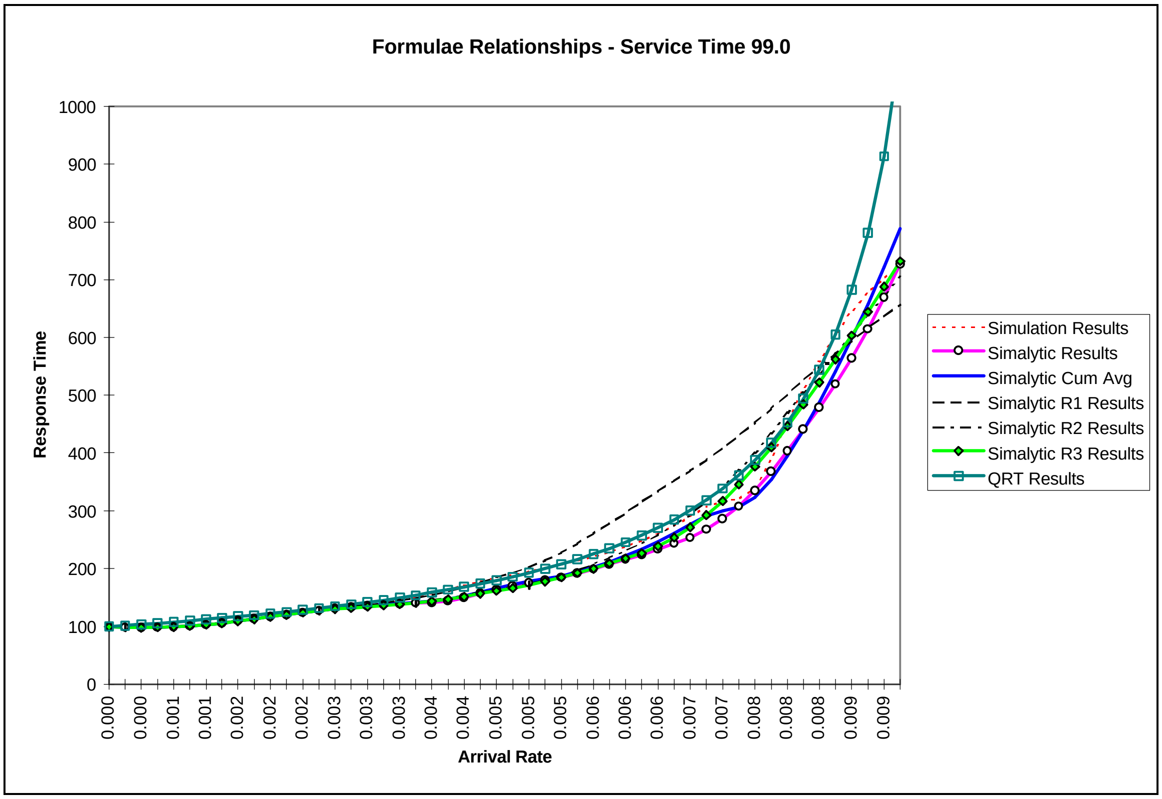

Formulae Relationships - Service Time 4.5
Response Time
Arrival Rate
0
10
20
30
40
50
60
0.002
0.011
0.019
0.028
0.037
0.045
0.054
0.063
0.071
0.080
0.089
0.097
0.106
0.115
0.123
0.132
0.141
0.149
0.158
0.167
0.175
0.184
0.193
0.201
0.210
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results


Formulae Relationships - Service Time 5.0
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results


Formulae Relationships - Service Time 25.0
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results


Formulae Relationships - Service Time 50.0
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results


Formulae Relationships - Service Time 75.0
Response Time
Arrival Rate
Simulation Results
Simalytic Results
Simalytic Cum Avg
Simalytic R1 Results
Simalytic R2 Results
Simalytic R3 Results
QRT Results



7.5.2 Relative Response Times

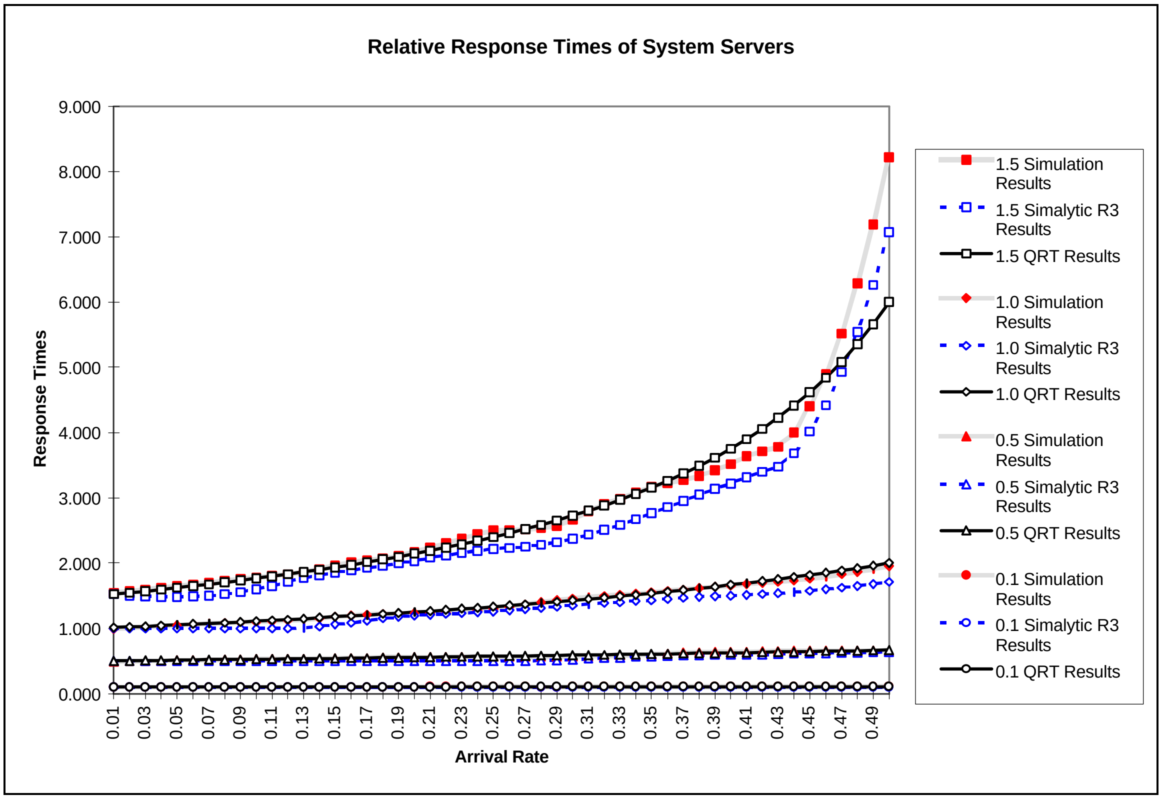
The next group of charts, labeled **Relative Response Times**, shows the resulting response times when servers are combined as discussed in sections *3.4.1.1* through *3.4.1.7*. These charts show 32 scenarios (*Table 1 Mathematical Formulae Results* on page 93 lists the scenarios), including both series and routing systems. Each chart, which contains either three or four related scenarios (on the same chart to reduce the total number of charts and pages required), shows how the Simalytic function results track the simulation results.

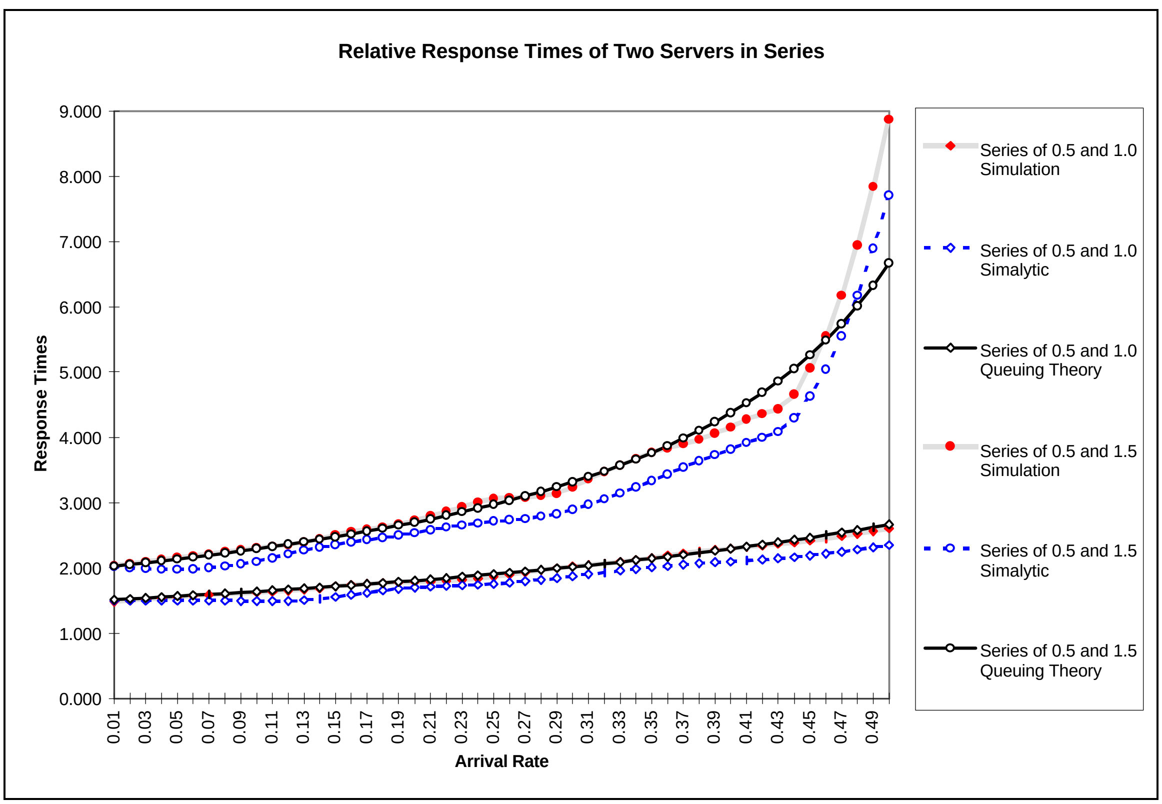
The first chart, **Relative Response Times of System Servers**, simply reproduces the results from the last charts in the same format, and using the same arrival rate scale, to provide easy reference between the single server results and the multi-server results. The lines are labeled with the server service time and the MathCAD result name (**Simulation**, **Simalytic R3** (as discussed above) and the queuing theory function **QRT**). For example, the bottom line in the legend is labeled **0.1 QRT Results** and represents the response time for a server with a service time of 0.1 calculated by the MathCAD formula (QueuingRT) for queuing theory response time in *Figure 59 MathCAD Worksheet for Queuing Theory Surface Plots* on page 178.

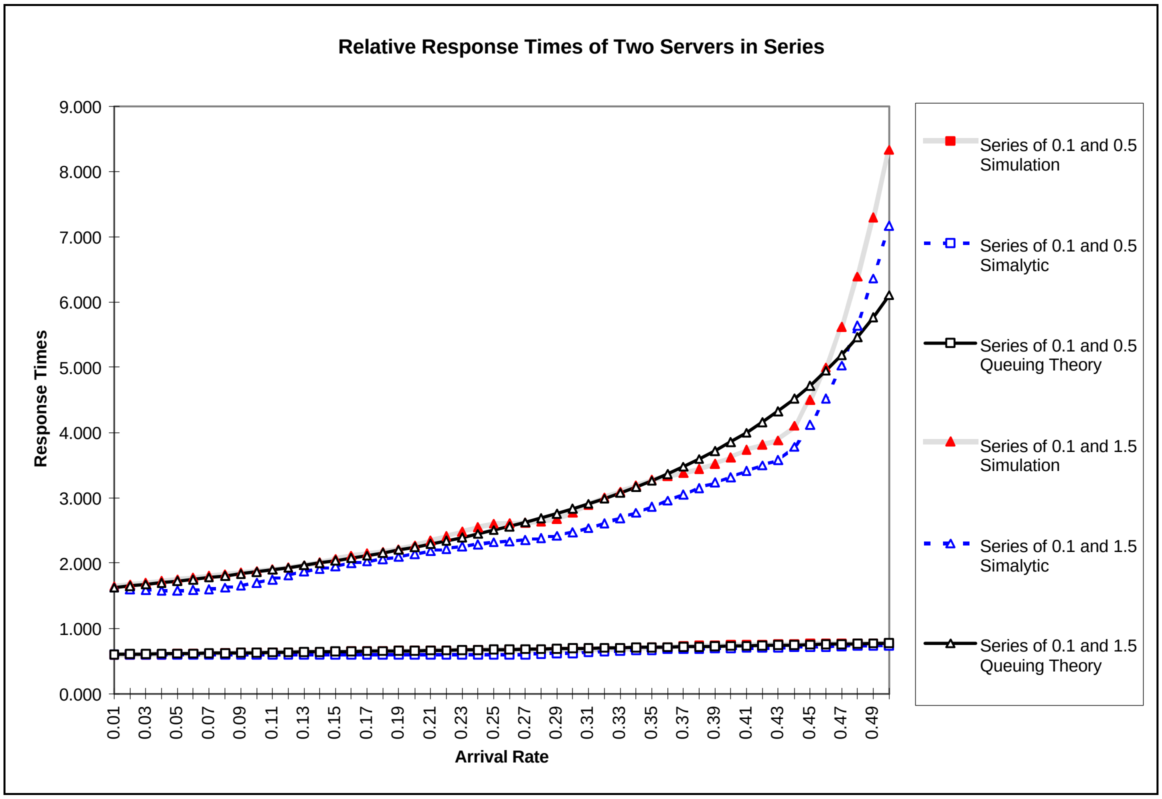
The rest of the charts show the resulting response times when servers are combined as discussed in sections *3.4.1.1* through *3.4.1.7*. Each line is labeled with the name of the scenario, which is intended to be descriptive of the number of servers, the relationship between servers (series or routing) and the service times of each of the servers. For example, in the chart titled **Relative Response Times of Two Servers in Series**, the top line in the legend is labeled, **Series of 0.1 and 0.5 Simulation,** and shows the results of combining two servers in series where one has a service time of 0.1 and the other has a service time of 0.5, and the results were created using the MathCAD simulation technique. The bottom line in the same legend, **Series of 0.5 and 1.5 Queuing Theory,** shows the results of combining two servers in series where one has a service time of 0.5 and the other has a service time of 1.5, and the results were created using the MathCAD queuing theory technique. Another example is shown in the chart titled **Relative Response Times of Routing to Two Servers 80% Short and 20% Long,** where the top line in the legend is labeled **Routing 80% 0.5 and 20% 1.5 Simulation,** and shows the results of combining two servers where 80% of the transactions are short (routed to the 0.5 service time server) and 20%

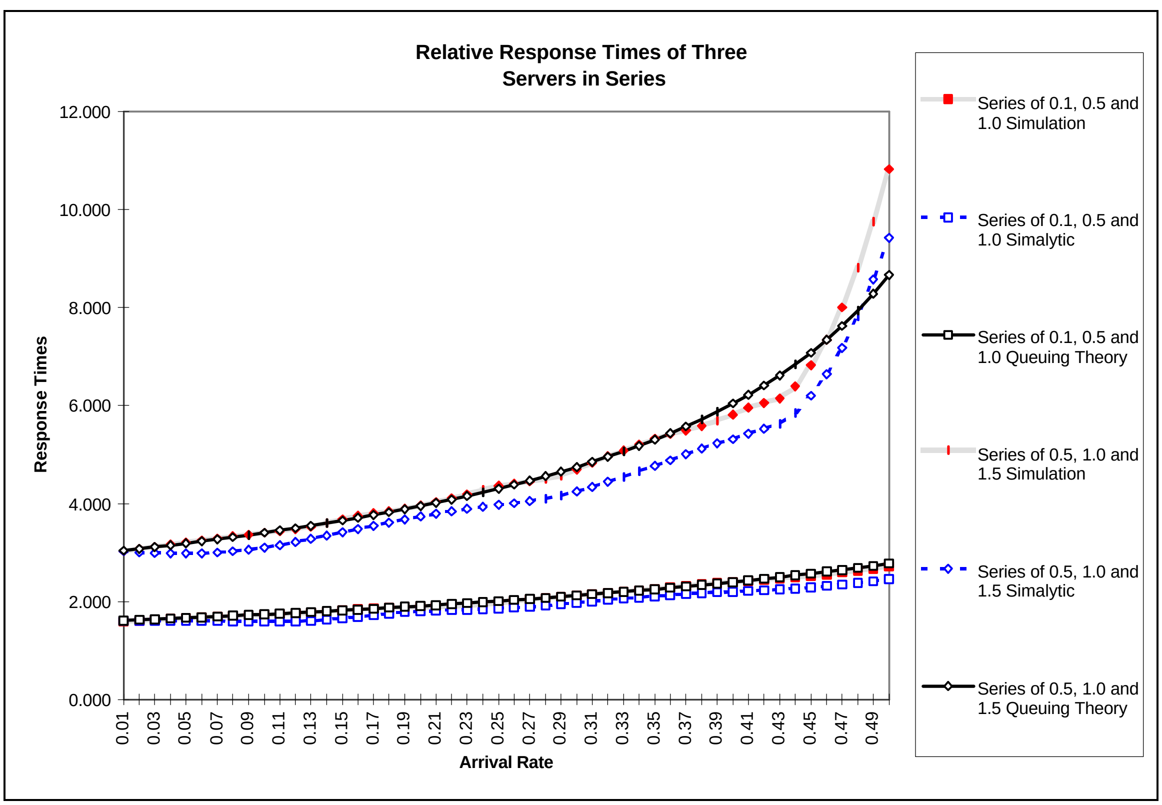
are long (routed to the 1.5 service time server) and the results were created using the MathCAD simulation technique.

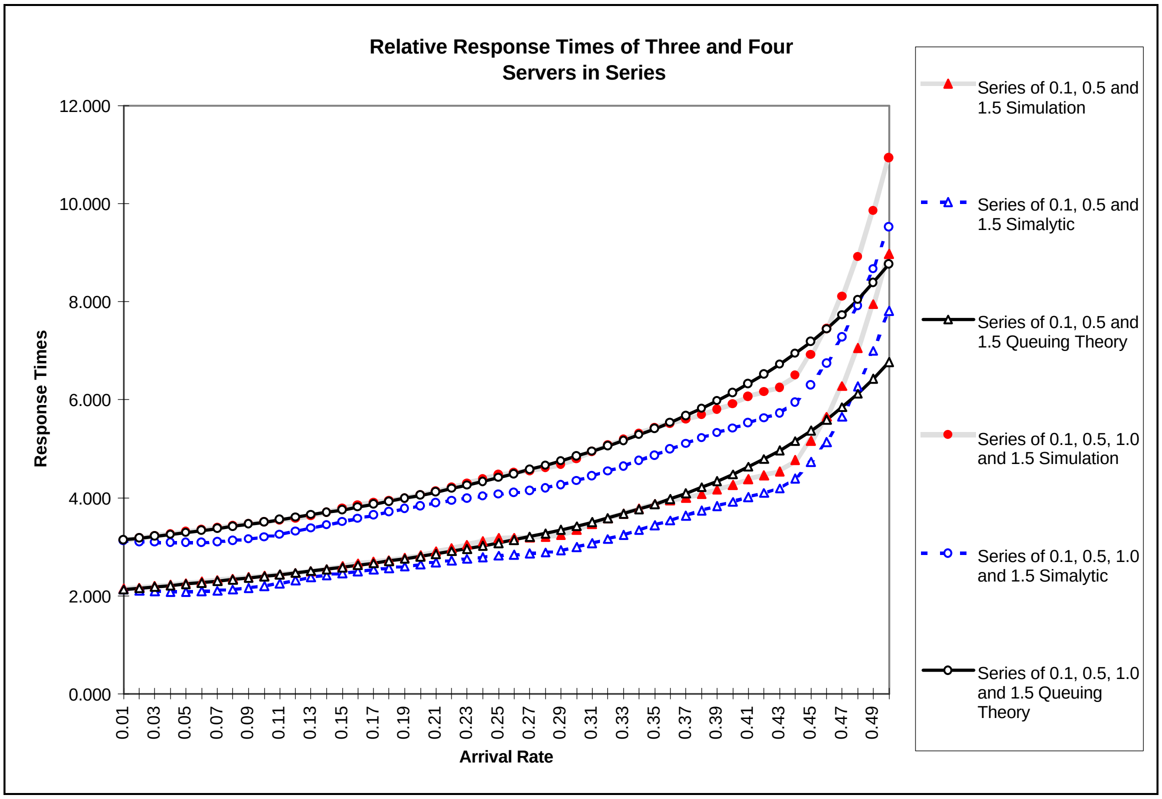
In theory, the lines with the same scenario name, except for the technique used, should produce the same curve (these have the same shaped symbol on each chart). For example, the lines labeled **Routing 80% 0.5 and 20% 1.5 Simulation, Routing 80% 0.5 and 20% 1.5 Simalytic** and **Routing 80% 0.5 and 20% 1.5 Queuing Theory,** (shown with a square symbol) should be indistinguishable. The small differences seen in the lines for each group show how well the results from the three techniques match. The purpose of these charts is to show that, in all cases, the differences are relatively small. Additional discussion and detailed analysis is in section *3.5.3 Validation of the Mathematical Foundation* on page 86.

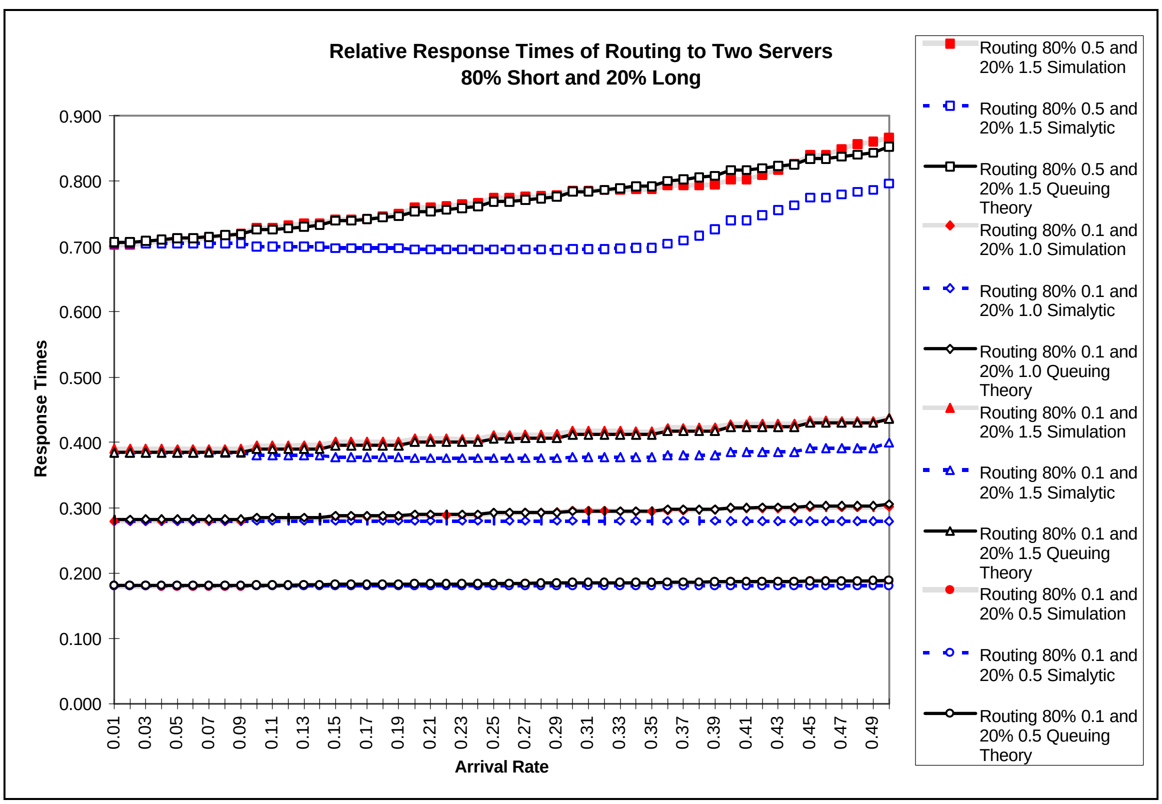

Relative Response Times of System Servers
Response Times
9.000
8.000
7.000
6.000
5.000
4.000
3.000
2.000
1.000
0.000
0.01
0.03
0.05
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
0.47
0.49
Arrival Rate
1.5 Simulation Results
1.5 Simalytic R3 Results
1.5 QRT Results
1.0 Simulation Results
1.0 Simalytic R3 Results
1.0 QRT Results
0.5 Simulation Results
0.5 Simalytic R3 Results
0.5 QRT Results
0.1 Simulation Results
0.1 Simalytic R3 Results
0.1 QRT Results

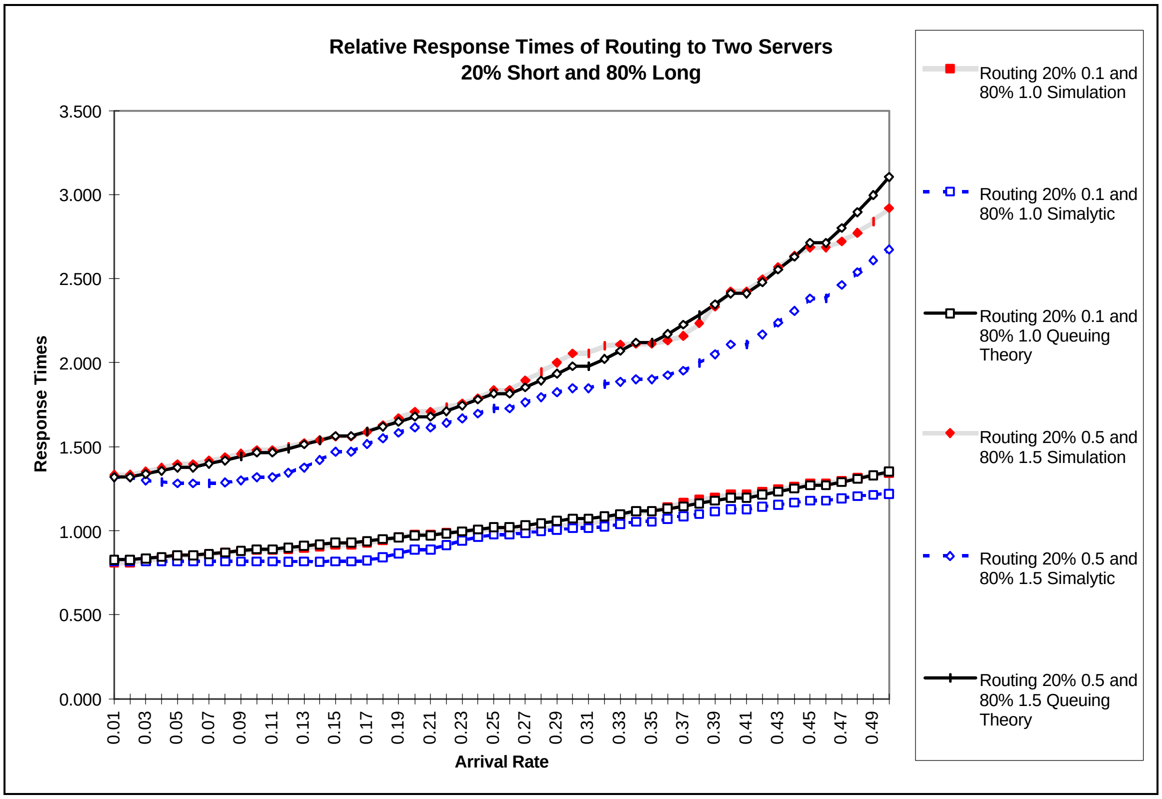

Relative Response Times of Two Servers in Series
Response Times
9.000
8.000
7.000
6.000
5.000
4.000
3.000
2.000
1.000
0.000
0.01
0.03
0.05
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
0.47
0.49
Arrival Rate
Series of 0.5 and 1.0 Simulation
Series of 0.5 and 1.0 Simalytic
Series of 0.5 and 1.0 Queuing Theory
Series of 0.5 and 1.5 Simulation
Series of 0.5 and 1.5 Simalytic
Series of 0.5 and 1.5 Queuing Theory


Relative Response Times of Two Servers in Series
Response Times
Arrival Rate
Series of 0.1 and 0.5 Simulation
Series of 0.1 and 0.5 Simalytic
Series of 0.1 and 0.5 Queuing Theory
Series of 0.1 and 1.5 Simulation
Series of 0.1 and 1.5 Simalytic
Series of 0.1 and 1.5 Queuing Theory

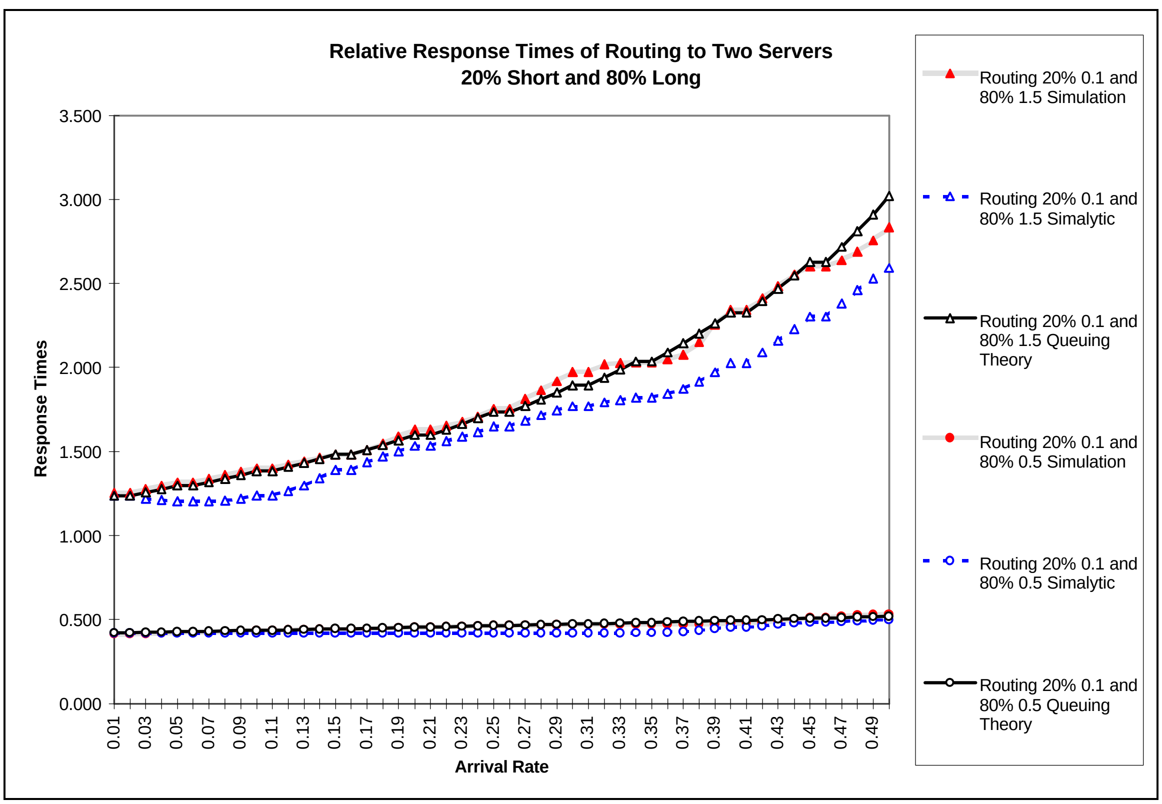

Relative Response Times of Three Servers in Series
Response Times
Arrival Rate
Series of 0.1, 0.5 and 1.0 Simulation
Series of 0.1, 0.5 and 1.0 Simalytic
Series of 0.1, 0.5 and 1.0 Queuing Theory
Series of 0.5, 1.0 and 1.5 Simulation
Series of 0.5, 1.0 and 1.5 Simalytic
Series of 0.5, 1.0 and 1.5 Queuing Theory


Relative Response Times of Three and Four Servers in Series
Response Times
Arrival Rate
Series of 0.1, 0.5 and 1.5 Simulation
Series of 0.1, 0.5 and 1.5 Simalytic
Series of 0.1, 0.5 and 1.5 Queuing Theory
Series of 0.1, 0.5, 1.0 and 1.5 Simulation
Series of 0.1, 0.5, 1.0 and 1.5 Simalytic
Series of 0.1, 0.5, 1.0 and 1.5 Queuing Theory

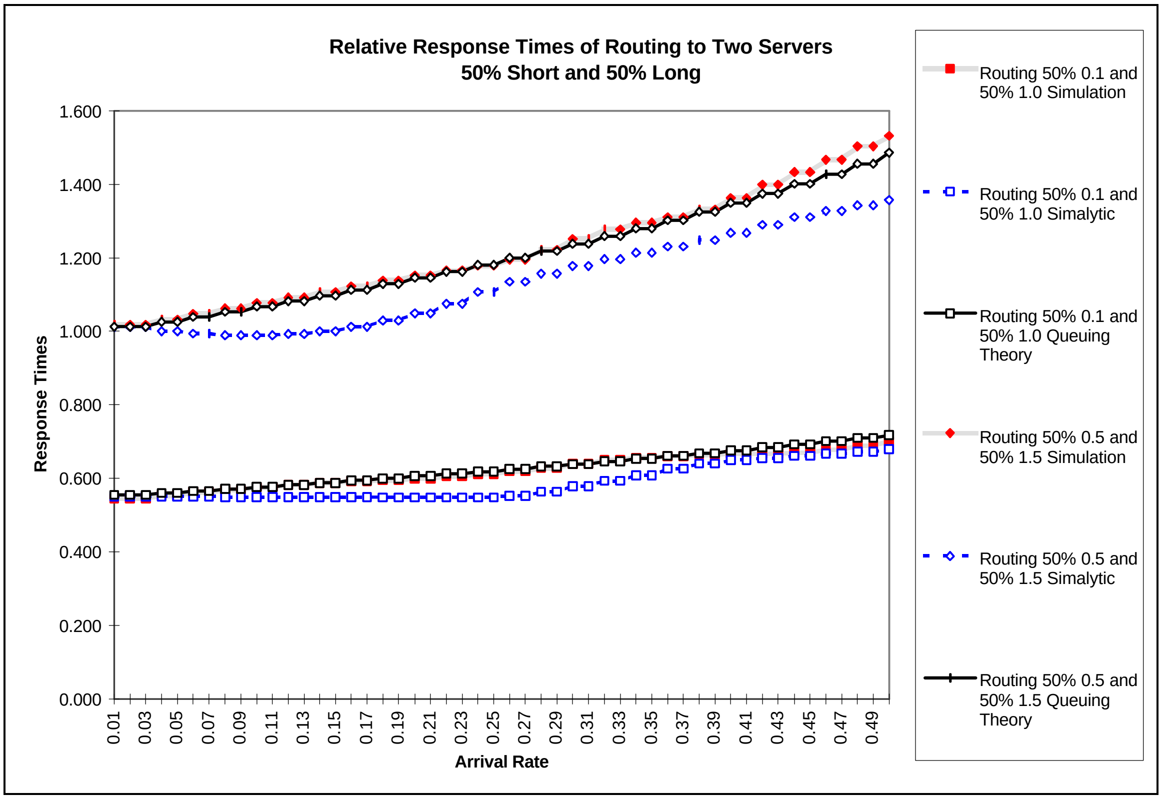

Relative Response Times of Routing to Two Servers 80% Short and 20% Long
Response Times
Arrival Rate
Routing 80% 0.5 and 20% 1.5 Simulation
Routing 80% 0.5 and 20% 1.5 Simalytic
Routing 80% 0.5 and 20% 1.5 Queuing Theory
Routing 80% 0.1 and 20% 1.0 Simulation
Routing 80% 0.1 and 20% 1.0 Simalytic
Routing 80% 0.1 and 20% 1.0 Queuing Theory
Routing 80% 0.1 and 20% 1.5 Simulation
Routing 80% 0.1 and 20% 1.5 Simalytic
Routing 80% 0.1 and 20% 1.5 Queuing Theory
Routing 80% 0.1 and 20% 0.5 Simulation
Routing 80% 0.1 and 20% 0.5 Simalytic
Routing 80% 0.1 and 20% 0.5 Queuing Theory

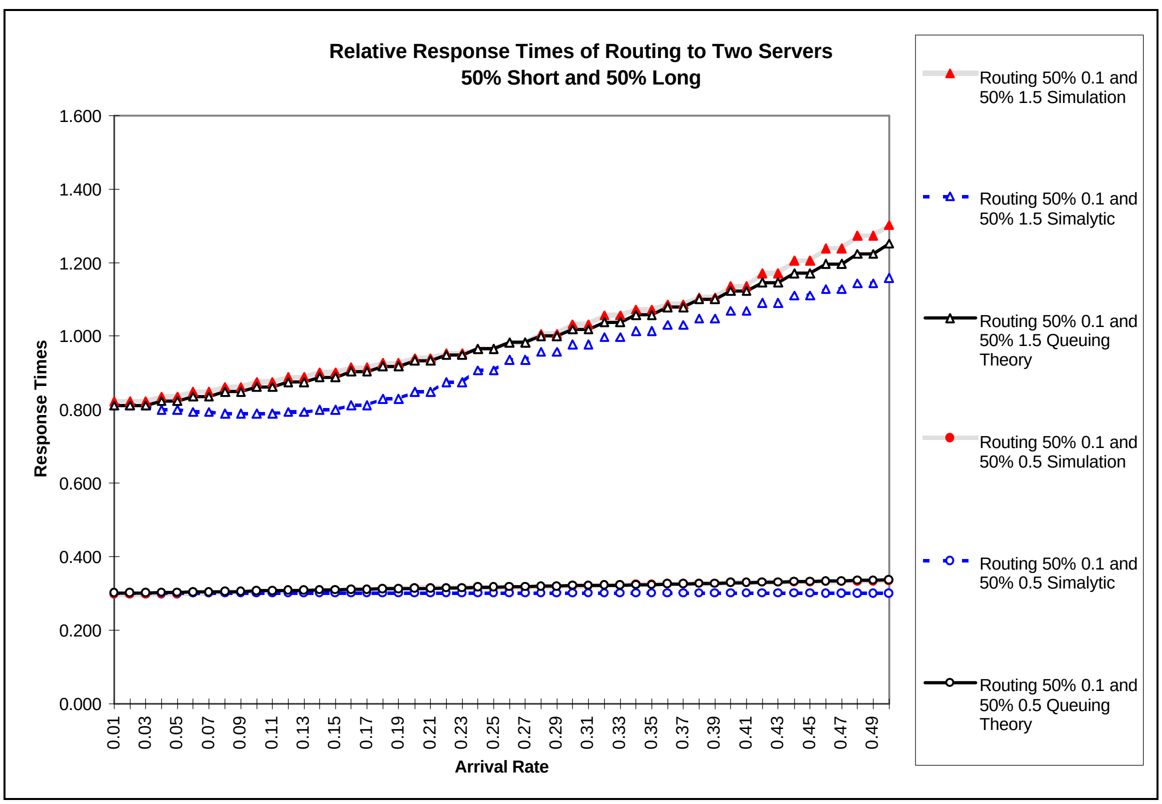


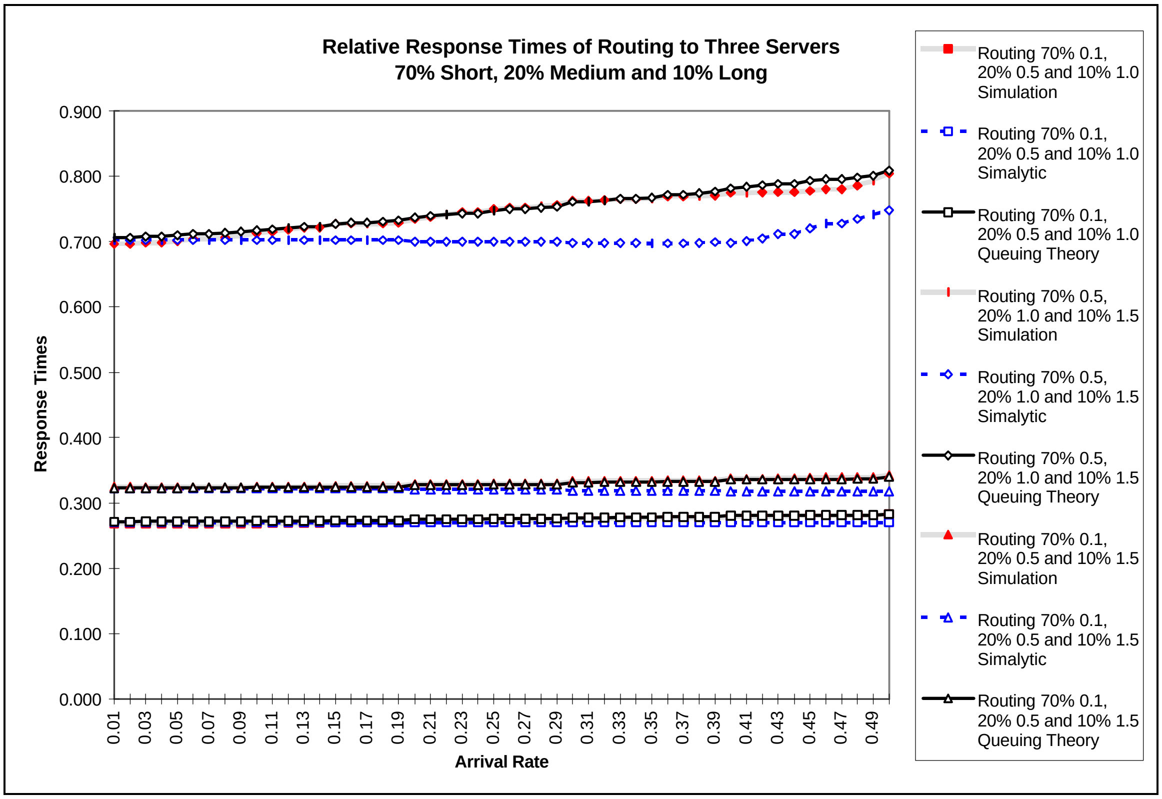
Relative Response Times of Routing to Two Servers
20% Short and 80% Long
Response Times
3.500
3.000
2.500
2.000
1.500
1.000
0.500
0.000
0.01
0.03
0.05
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
0.47
0.49
Arrival Rate
Routing 20% 0.1 and 80% 1.0 Simulation
Routing 20% 0.1 and 80% 1.0 Simalytic
Routing 20% 0.1 and 80% 1.0 Queuing Theory
Routing 20% 0.5 and 80% 1.5 Simulation
Routing 20% 0.5 and 80% 1.5 Simalytic
Routing 20% 0.5 and 80% 1.5 Queuing Theory



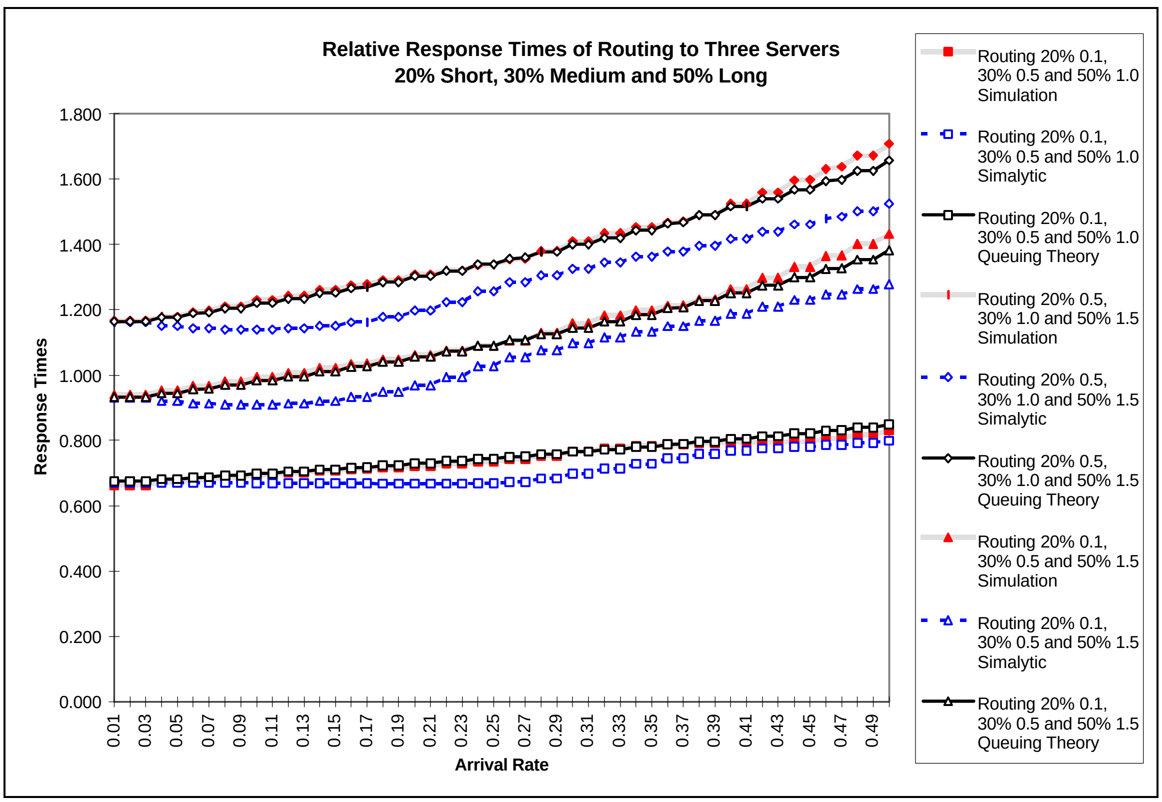
Relative Response Times of Routing to Two Servers
20% Short and 80% Long
Response Times
3.500
3.000
2.500
2.000
1.500
1.000
0.500
0.000
0.01
0.03
0.05
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
0.47
0.49
Arrival Rate
Routing 20% 0.1 and 80% 1.5 Simulation
Routing 20% 0.1 and 80% 1.5 Simalytic
Routing 20% 0.1 and 80% 1.5 Queuing Theory
Routing 20% 0.1 and 80% 0.5 Simulation
Routing 20% 0.1 and 80% 0.5 Simalytic
Routing 20% 0.1 and 80% 0.5 Queuing Theory


Relative Response Times of Routing to Two Servers
50% Short and 50% Long
Response Times
Arrival Rate
Routing 50% 0.1 and 50% 1.0 Simulation
Routing 50% 0.1 and 50% 1.0 Simalytic
Routing 50% 0.1 and 50% 1.0 Queuing Theory
Routing 50% 0.5 and 50% 1.5 Simulation
Routing 50% 0.5 and 50% 1.5 Simalytic
Routing 50% 0.5 and 50% 1.5 Queuing Theory

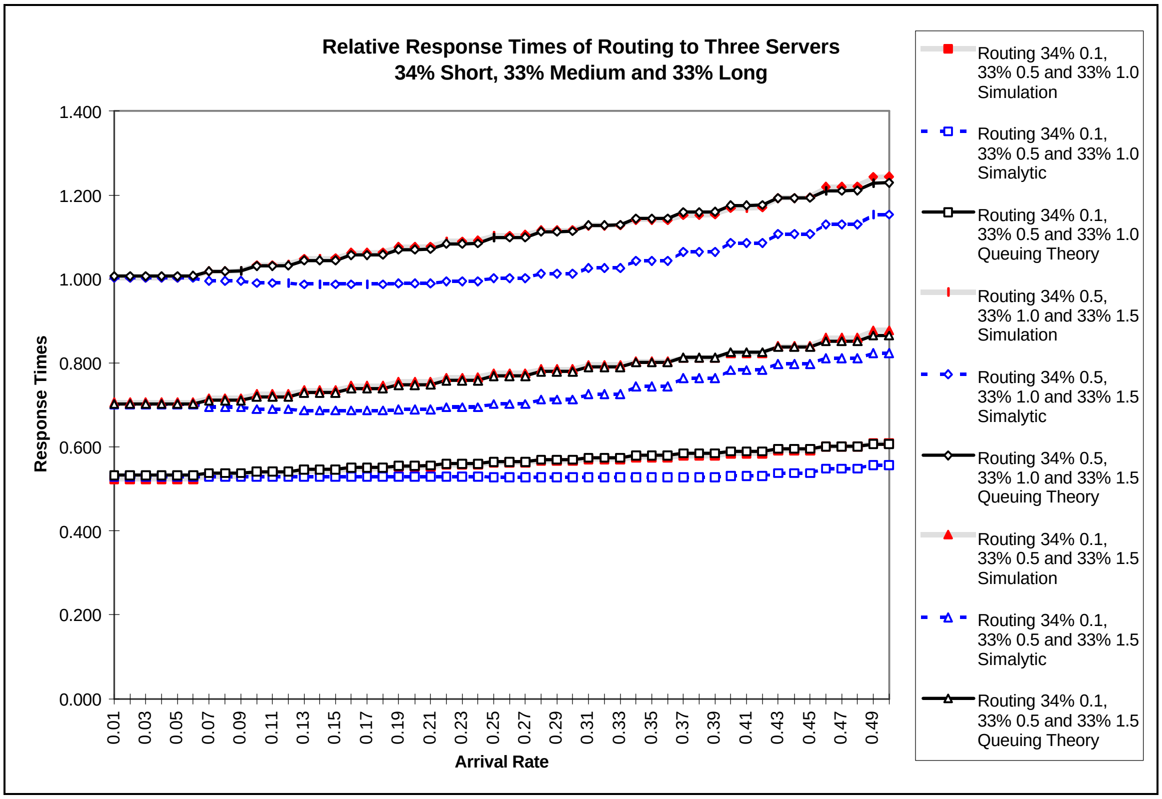

Relative Response Times of Routing to Two Servers
50% Short and 50% Long
Response Times
Arrival Rate
Routing 50% 0.1 and 50% 1.5 Simulation
Routing 50% 0.1 and 50% 1.5 Simalytic
Routing 50% 0.1 and 50% 1.5 Queuing Theory
Routing 50% 0.1 and 50% 0.5 Simulation
Routing 50% 0.1 and 50% 0.5 Simalytic
Routing 50% 0.1 and 50% 0.5 Queuing Theory

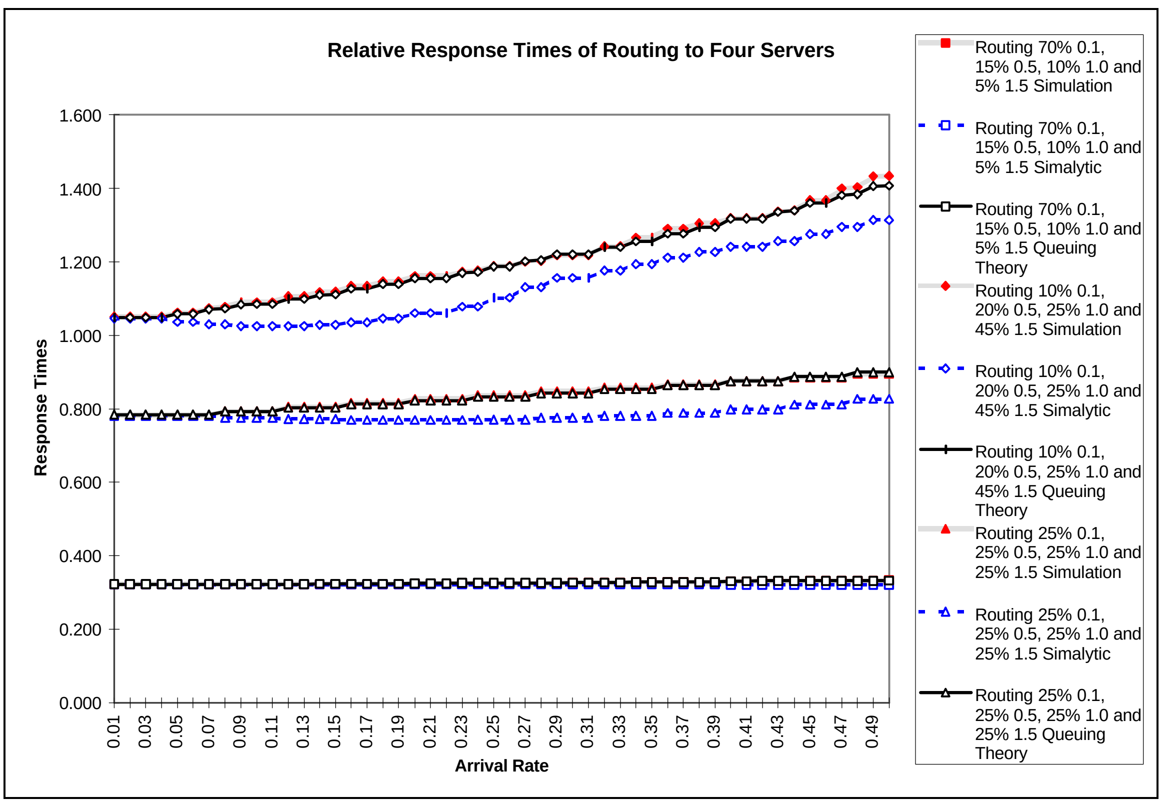


Relative Response Times of Routing to Three Servers
70% Short, 20% Medium and 10% Long
Response Times
Arrival Rate
0.900
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
0.01
0.03
0.05
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
0.47
0.49
Routing 70% 0.1, 20% 0.5 and 10% 1.0 Simulation
Routing 70% 0.1, 20% 0.5 and 10% 1.0 Simalytic
Routing 70% 0.1, 20% 0.5 and 10% 1.0 Queuing Theory
Routing 70% 0.5, 20% 1.0 and 10% 1.5 Simulation
Routing 70% 0.5, 20% 1.0 and 10% 1.5 Simalytic
Routing 70% 0.5, 20% 1.0 and 10% 1.5 Queuing Theory
Routing 70% 0.1, 20% 0.5 and 10% 1.5 Simulation
Routing 70% 0.1, 20% 0.5 and 10% 1.5 Simalytic
Routing 70% 0.1, 20% 0.5 and 10% 1.5 Queuing Theory



Relative Response Times of Routing to Three Servers
20% Short, 30% Medium and 50% Long
Response Times
Arrival Rate
1.800
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0.000
0.01
0.03
0.05
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
0.47
0.49
Routing 20% 0.1, 30% 0.5 and 50% 1.0 Simulation
Routing 20% 0.1, 30% 0.5 and 50% 1.0 Simalytic
Routing 20% 0.1, 30% 0.5 and 50% 1.0 Queuing Theory
Routing 20% 0.5, 30% 1.0 and 50% 1.5 Simulation
Routing 20% 0.5, 30% 1.0 and 50% 1.5 Simalytic
Routing 20% 0.5, 30% 1.0 and 50% 1.5 Queuing Theory
Routing 20% 0.1, 30% 0.5 and 50% 1.5 Simulation
Routing 20% 0.1, 30% 0.5 and 50% 1.5 Simalytic
Routing 20% 0.1, 30% 0.5 and 50% 1.5 Queuing Theory


Relative Response Times of Routing to Three Servers
34% Short, 33% Medium and 33% Long
Response Times
Arrival Rate
Routing 34% 0.1, 33% 0.5 and 33% 1.0 Simulation
Routing 34% 0.1, 33% 0.5 and 33% 1.0 Simalytic
Routing 34% 0.1, 33% 0.5 and 33% 1.0 Queuing Theory
Routing 34% 0.5, 33% 1.0 and 33% 1.5 Simulation
Routing 34% 0.5, 33% 1.0 and 33% 1.5 Simalytic
Routing 34% 0.5, 33% 1.0 and 33% 1.5 Queuing Theory
Routing 34% 0.1, 33% 0.5 and 33% 1.5 Simulation
Routing 34% 0.1, 33% 0.5 and 33% 1.5 Simalytic
Routing 34% 0.1, 33% 0.5 and 33% 1.5 Queuing Theory


Relative Response Times of Routing to Four Servers
Response Times
Arrival Rate
Routing 70% 0.1, 15% 0.5, 10% 1.0 and 5% 1.5 Simulation
Routing 70% 0.1, 15% 0.5, 10% 1.0 and 5% 1.5 Simalytic
Routing 70% 0.1, 15% 0.5, 10% 1.0 and 5% 1.5 Queuing Theory
Routing 10% 0.1, 20% 0.5, 25% 1.0 and 45% 1.5 Simulation
Routing 10% 0.1, 20% 0.5, 25% 1.0 and 45% 1.5 Simalytic
Routing 10% 0.1, 20% 0.5, 25% 1.0 and 45% 1.5 Queuing Theory
Routing 25% 0.1, 25% 0.5, 25% 1.0 and 25% 1.5 Simulation
Routing 25% 0.1, 25% 0.5, 25% 1.0 and 25% 1.5 Simalytic
Routing 25% 0.1, 25% 0.5, 25% 1.0 and 25% 1.5 Queuing Theory

```
vsize, msize number of transactions     <== set by user
ServiceTime  average server service time      <== set by user

IRT           an array containing the interarrival times between each
              transaction
Arv           an array containing the arrival time for each transaction
Dep           an array containing the departure time for each transaction
Srv           an array containing the service time for each transaction
              (exponential distribution)
SimulationResp      an array containing the response time for each tran saction
              (service + wait time)
SRTimes       an array containing the response times for each transaction from
              the Simalytic function

rexp(x,y)     Built-in exponential function that returns an array of x elements
              with rate y.
QRT(r)        Queuing theory function returns response time for rate r and
              ServiceTime.
SRT(r)        Simulation function returns response time for rate r and
              ServiceTime.
Simalytic(ir)       Transform function that returns response time for
              interarrival rate ir and ServiceTime.

Matrix indices are: i, n, r, m, m_i, m_n

Each of the major arrays (above) is a single column.  The same names are used
prefixed with a lower case 'a' to show a multi-column version of the array
(i.e. IRT and aIRT).

Variables used to calculate the steps in the Simalytic function SrvRTime:
I1, I2, I3, I4,  I5, I6, I7, I8 and I9 calculate the interarrival times for
              the steps.
S1, S2, S3, S4,  S5, S6, S7, S8 and S9 calculate the response times for the
              steps.

The arrays vs1, vx and MaResp are used with the built-in functions loess and
interp to fit a curve to the results of the simulation.

msize is used for the simulation matrix to get a larger number to sample (same
              usage a vsize).
```

$ServiceTime := 5.0$ **Service rate for this analysis run.**

$vsize := 1000$ **Numbers of transactions for this analysis run.**

rate is calculated as 1/ServiceTime to set the upper bound at the saturation point.

$$rate := \frac{1}{ServiceTime} - .01$$

Define a queuing theory function to calculate the response time when given an arrival rate. The service time is a constant for this analysis run. (The function only returns positive response times. The calculated time goes negative when the server is saturated and the function returns infinity to show this. Because of MathCAD rounding of intermediate values a small value is added to avoid a divide by zero error.)

$$QRT(q\_rate) := \left| \begin{array}{l} rt \leftarrow \dfrac{ServiceTime}{(1 - q\_rate \cdot ServiceTime) + .00000001} \\ rt \ \text{ if } \ rt > 0 \\ \infty \ \text{ if } \ rt \le 0 \\ \infty \ \text{ otherwise} \end{array} \right.$$

Figure 60 MathCAD Worksheet Listing

Set matrix indices (MathCAD Matrices are zero based)

$i := 0 .. vsize - 1$

$n := 1 .. vsize - 1$ **(i for all elements, n for all but first)**

$r := .001, .002 .. rate$ **(r sets the arrival rates to use for plots)**

Create arrays of interarrival times (IRT) and service times (Srv)

$IRT := rexp(vsize, rate)$ $Srv := rexp(vsize, rate)$

Create a series of step points for the transform function (interarrival times for step points)

$I1 := \frac{1}{rate} \cdot 6$ $I2 := \frac{1}{rate} \cdot 4$ $I3 := \frac{1}{rate} \cdot 3$ $I4 := \frac{1}{rate} \cdot 2$ $I5 := \frac{1}{rate} \cdot 1.5$

$I6 := \frac{1}{rate} \cdot 1.25$ $I7 := \frac{1}{rate} \cdot 1.15$ $I8 := \frac{1}{rate} \cdot 1.1$ $I9 := \frac{1}{rate} \cdot 1.02$

(response times for step points)

$S1 := QRT\left(\frac{1}{I1}\right)$ $S2 := QRT\left(\frac{1}{I2}\right)$ $S3 := QRT\left(\frac{1}{I3}\right)$ $S4 := QRT\left(\frac{1}{I4}\right)$ $S5 := QRT\left(\frac{1}{I5}\right)$

$S6 := QRT\left(\frac{1}{I6}\right)$ $S7 := QRT\left(\frac{1}{I7}\right)$ $S8 := QRT\left(\frac{1}{I8}\right)$ $S9 := QRT\left(\frac{1}{I9}\right)$

(cumlative average index, ca_i=initital part, ca=main part)

$Cavg_i := mean(submatrix(IRT, 0, i, 0, 0))$ **(cumlative average of all events)**

(initialize the first part of the array to low ir value to represent warm-up interval)

$Cavg_{ca\_i} := I1 \cdot 1.1$

$ra1 := 5$ $rai1 := ra1 .. vsize - 1$ $rai1\_i := 0 .. ra1 - 1$

$ra2 := 10$ $rai2 := ra2 .. vsize - 1$ $rai2\_i := 0 .. ra2 - 1$

$ra3 := 25$ $rai3 := ra3 .. vsize - 1$ $rai3\_i := 0 .. ra3 - 1$

$Ravg1_{rai1} := mean(submatrix(IRT, rai1 - ra1, rai1, 0, 0))$

$Ravg1_{rai1\_i} := Ravg1_{ra1}$ **(rolling average of ra1 events)**

$Ravg2_{rai2} := mean(submatrix(IRT, rai2 - ra2, rai2, 0, 0))$

$Ravg2_{rai2\_i} := Ravg2_{ra2}$ **(rolling average of ra2 events)**

$Ravg3_{rai3} := mean(submatrix(IRT, rai3 - ra3, rai3, 0, 0))$

$Ravg3_{rai3\_i} := Ravg3_{ra3}$ **(rolling average of ra3 events)**

Define the transform function using the step point rates and service times

$$Simalytic(ir) := \begin{cases} ServiceTime & \text{if } I1 < ir \\ S1 & \text{if } I2 < ir \le I1 \\ S2 & \text{if } I3 < ir \le I2 \\ S3 & \text{if } I4 < ir \le I3 \\ S4 & \text{if } I5 < ir \le I4 \\ S5 & \text{if } I6 < ir \le I5 \\ S6 & \text{if } I7 < ir \le I6 \\ S7 & \text{if } I8 < ir \le I7 \\ S8 & \text{if } I9 < ir \le I8 \\ S9 & \text{otherwise} \end{cases}$$

Figure 60 continued from previous page

Create arrays of transaction arrival times (Arv), transaction departure times (Dep) and transaction response times (SimulationResp) to simulate the system. This technique is taken from the MathCAD Statistics Electronic Book, Simulation section.

$Arv_0 := 0$ **(the first arrival time is zero)**

(each following time adds an interarrival time to the prior arrival time)

$Arv_n := Arv_{n-1} + IRT_{n-1}$

$Dep_0 := Arv_0 + Srv_0$ **(the first departure time is the first arrival time plus the service time)**

$Dep_n := \text{if}\left(Arv_n > Dep_{n-1}, Arv_n + Srv_n, Dep_{n-1} + Srv_n\right)$

(each following time is either the arrival time plus the service time (no queue) or the prior departure time plus the service time. the response times are the differences between transaction arrival times and departure times)

$SimulationResp := Dep - Arv$

$MaxATime := Arv_{vsize-1}$

$MaxDTime := Dep_{vsize-1}$

$ARate := \frac{vsize}{MaxATime}$

Create an array containing the response times for each transaction from the Simalytic function.

$SRTimes_n := Simalytic\left(IRT_n\right)$

$QSRTimes_n := QRT\left(\frac{1}{IRT_n}\right)$

$SR\_Ravg1_n := Simalytic\left(Ravg1_n\right)$

$SR\_Cavg_n := Simalytic\left(Cavg_n\right)$

$SR\_Ravg2_n := Simalytic\left(Ravg2_n\right)$

$SR\_Ravg3_n := Simalytic\left(Ravg3_n\right)$

Results are generated from a series of simulations at different arrival rates. The simulation builds columns of the array of response times at m different arrival rates between .1 and rate+.1. The average for each column is then plotted and a curve fit to the points. That curve is then laid on top of the queuing theory and Simalytic function curves from above.

(current values)

$msize := 1000$ **(number of columns in the matrix - similar to vsize above)**

$m\_i := 0 .. msize - 1$

$m\_n := 1 .. msize - 1$ **(indices for matrix)**

(create an array of arrival rates)

$max\_m := 49$ **(number of columns on the matrix)**

$m := 0 .. max\_m$ **(columns index for matrix)**

$mrate_m := rate \cdot \left(\frac{m}{max\_m} + .01\right)$

$z_m := 0$ **(set a temporary vector to zeros)**

Figure 60 continued from previous page

Set each column to be an array of exponential values

$aIRT^{<m>} := rexp(msize, mrate_m)$ **(array of interarrival times at different means)**

$MaIRT_m := mean(aIRT^{<m>})$

$aSrv^{<m>} := rexp\left(msize, \frac{1}{ServiceTime}\right)$ **(array of service times at the same mean)**

$MaSrv_m := mean(aSrv^{<m>})$

$aArv^{<0>} := z$

$aArv_{m\_n, m} := aArv_{m\_n-1, m} + aIRT_{m\_n-1, m}$ **(set the arrival times for each column)**

$aDep_{0,m} := aArv_{0,m} + aSrv_{0,m}$ **(set the departure times for each column)**

$aDep_{m\_n, m} := if(aArv_{m\_n, m} > aDep_{m\_n-1, m}, aArv_{m\_n, m} + aSrv_{m\_n, m}, aDep_{m\_n-1, m} + aSrv_{m\_n, m})$

$aSimulationResp := aDep - aArv$ **(set the response times for each column)**

$MaResp_m := mean(aSimulationResp^{<m>})$ **(create a vector of the average response time of each column)**

$mra1 := 15$

$mrai1 := mra1 .. msize - 1$

$mrai1\_i := 0 .. mra1 - 1$

$mra2 := 50$

$mrai2 := mra2 .. msize - 1$

$mrai2\_i := 0 .. mra2 - 1$

$mra3 := 100$

$mrai3 := mra3 .. msize - 1$

$mrai3\_i := 0 .. mra3 - 1$

$aCavg_{m\_i, m} := mean(submatrix(aIRT^{<m>}, 0, m\_i, 0, 0))$

$aRavg1_{mrai1, m} := mean(submatrix(aIRT^{<m>}, mrai1 - mra1, mrai1, 0, 0))$

$aRavg1_{mrai1\_i, m} := aRavg1_{mra1, m}$

$aRavg2_{mrai2, m} := mean(submatrix(aIRT^{<m>}, mrai2 - mra2, mrai2, 0, 0))$

$aRavg2_{mrai2\_i, m} := aRavg2_{mra2, m}$

$aRavg3_{mrai3, m} := mean(submatrix(aIRT^{<m>}, mrai3 - mra3, mrai3, 0, 0))$

$aRavg3_{mrai3\_i, m} := aRavg3_{mra3, m}$

$aSRTimesC_{m\_i, m} := Simalytic(aCavg_{m\_i, m})$

$aSRTimesR1_{m\_i, m} := Simalytic(aRavg1_{m\_i, m})$

$aSRTimesR2_{m\_i, m} := Simalytic(aRavg2_{m\_i, m})$

$aSRTimesR3_{m\_i, m} := Simalytic(aRavg3_{m\_i, m})$

$MaSimalyticResp_m := Simalytic(MaIRT_m)$

$MaSimalyticRespC_m := mean(aSRTimesC^{<m>})$

$MaSimalyticRespR1_m := mean(aSRTimesR1^{<m>})$

$MaSimalyticRespR2_m := mean(aSRTimesR2^{<m>})$

$MaSimalyticRespR3_m := mean(aSRTimesR3^{<m>})$

Figure 60 continued from previous page

The MathCAD built-in functions (loess and interp) are used to fit a line to the average response times of each column.

```
vx_m := m
vs1 := loess(vx, MaResp, .35)
vs2 := loess(vx, MaSimalyticResp, .35)
SimuResults(x) := interp(vs1, vx, MaResp, vx_m)
SimalyResults(x) := interp(vs2, vx, MaSimalyticResp, vx_m)
SimulationResults_m := SimuResults(1)
SimalyticResults_m := SimalyResults(1)
vs2C := loess(vx, MaSimalyticRespC, .35)
vs2R1 := loess(vx, MaSimalyticRespR1, .35)
SimalyResultsC(x) := interp(vs2C, vx, MaSimalyticRespC, vx_m)
SimalyResultsR1(x) := interp(vs2R1, vx, MaSimalyticRespR1, vx_m)
SimalyticResultsC_m := SimalyResultsC(1)
SimalyticResultsR1_m := SimalyResultsR1(1)
vs2R3 := loess(vx, MaSimalyticRespR3, .35)
vs2R2 := loess(vx, MaSimalyticRespR2, .35)
SimalyResultsR3(x) := interp(vs2R3, vx, MaSimalyticRespR3, vx_m)
SimalyResultsR2(x) := interp(vs2R2, vx, MaSimalyticRespR2, vx_m)
SimalyticResultsR3_m := SimalyResultsR3(1)
SimalyticResultsR2_m := SimalyResultsR2(1)
```

Write the results to file to be imported into Excel.

```
WRITE(McadData) := ServiceTime
APPEND(McadData) := ArrivalRate
WRITEPRN(McadD_U) := SimulationResults
WRITEPRN(McadD_Y) := SimalyticResults
WRITEPRN(McadD_YC) := SimalyticResultsC
WRITEPRN(McadD_YR1) := SimalyticResultsR1
WRITEPRN(McadD_YR2) := SimalyticResultsR2
WRITEPRN(McadD_YR3) := SimalyticResultsR3
WRITEPRN(McadD_mrate) := mrate
WRITEPRN(McadD_QT) := QRT(mrate_m)
```

Figure 60 continued from previous page

7.6 Appendix F: MathCAD/Simul8 Comparison Results

The charts in this section show the relationship between the Simul8 results and the MathCAD results, comparing the two modeling implementations. The charts show results for the scenarios in *Table 11 MathCAD/Simul8 Results Scenarios*. Each scenario is identified by the number of servers and the service time for each. The routing scenarios also indicate the percent of the transactions routed to each server. Each scenario was modeled at arrival rates of 0.1, 0.2, 0.3, 0.4, 0.45 and 0.5 transactions per second.

The charts beginning on page 207 show the relationship between the MathCAD models and the Simul8 models. A blank space separates each group of results for a given number of servers (one, two, three, or four). Each group of three vertical bars represents the results of one modeling scenario implemented with Simul8. There are three bars because

Series Scenarios		Routing Scenarios
Single server		**Two servers**
0.1		80% 0.1 & 20% 1.0
0.5		80% 0.5 & 20% 1.5
1.0		80% 0.1 & 20% 1.5
1.5		80% 0.1 & 20% 0.5
		20% 0.1 & 80% 1.0
Two servers		20% 0.5 & 80% 1.5
0.1 & 0.5		20% 0.1 & 80% 1.5
0.5 & 1.0		20% 0.1 & 80% 0.5
0.1 & 1.5		50% 0.1 & 50% 1.0
0.5 & 1.5		50% 0.5 & 50% 1.5
		50% 0.1 & 50% 1.5
Three servers		50% 0.1 & 50% 0.5
0.1, 0.5 & 1.0		
0.1, 0.5 & 1.5		**Three servers**
0.5, 1.0 & 1.5		70% 0.1, 20% 0.5 & 10% 1.0
		70% 0.5, 20% 1.0 & 10% 1.5
Four servers		70% 0.1, 20% 0.5 & 10% 1.5
0.1, 0.5, 1.0 & 1.5		20% 0.1, 30% 0.5 & 50% 1.0
		20% 0.5, 30% 1.0 & 50% 1.5
High service time		20% 0.1, 30% 0.5 & 50% 1.5
25.0		34% 0.1, 33% 0.5 & 33% 1.0
50.0		34% 0.5, 33% 1.0 & 33% 1.5
75.0		34% 0.1, 33% 0.5 & 33% 1.5
99.0		
		Four servers
		70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
		10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
		25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5

Table 11 MathCAD/Simul8 Results Scenarios

Simul8 produces response time results for the ±95% confidence limits in addition to an average. The line on the chart represents the results of the MathCAD implementation from *Table 1 Mathematical Formulae Results* on page 93. The format of the charts is somewhat non-conventional in that the line does not represent a continuous series of related values. A line was used to connect the independent values for several reasons: to allow the reader to easily find each data point, to visually distinguish the MathCAD results from the Simul8 results, to provide an overall impression of the shape of the MathCAD results and to allow easy comparison between the two different groups of data points.

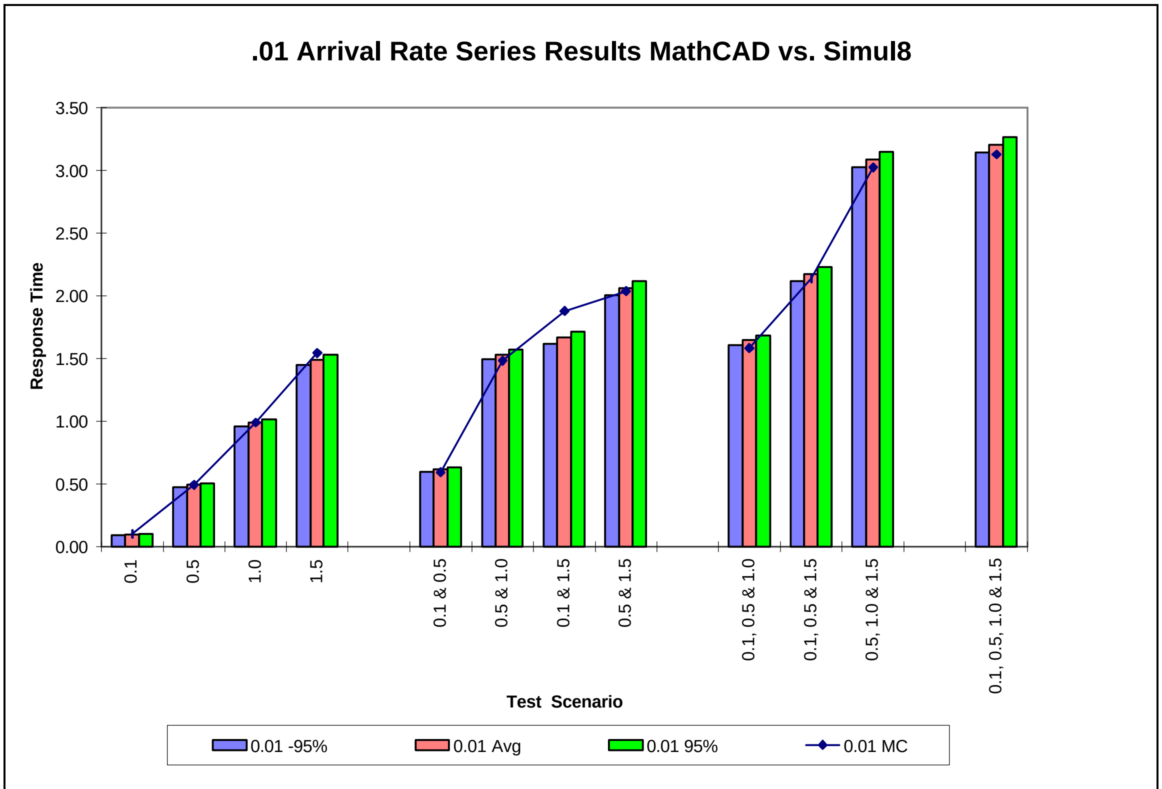
Arrival Rates	**Service Times**			
	25.0	**50.0**	**75.0**	**99.0**
0.001	✓	✓	✓	✓
0.005	✓	✓	✓	✓
0.009	✓	✓	✓	✓
0.0125	✓	✓	✓	
0.015	✓	✓		
0.02	✓	✓		
0.03	✓			
0.04	✓			

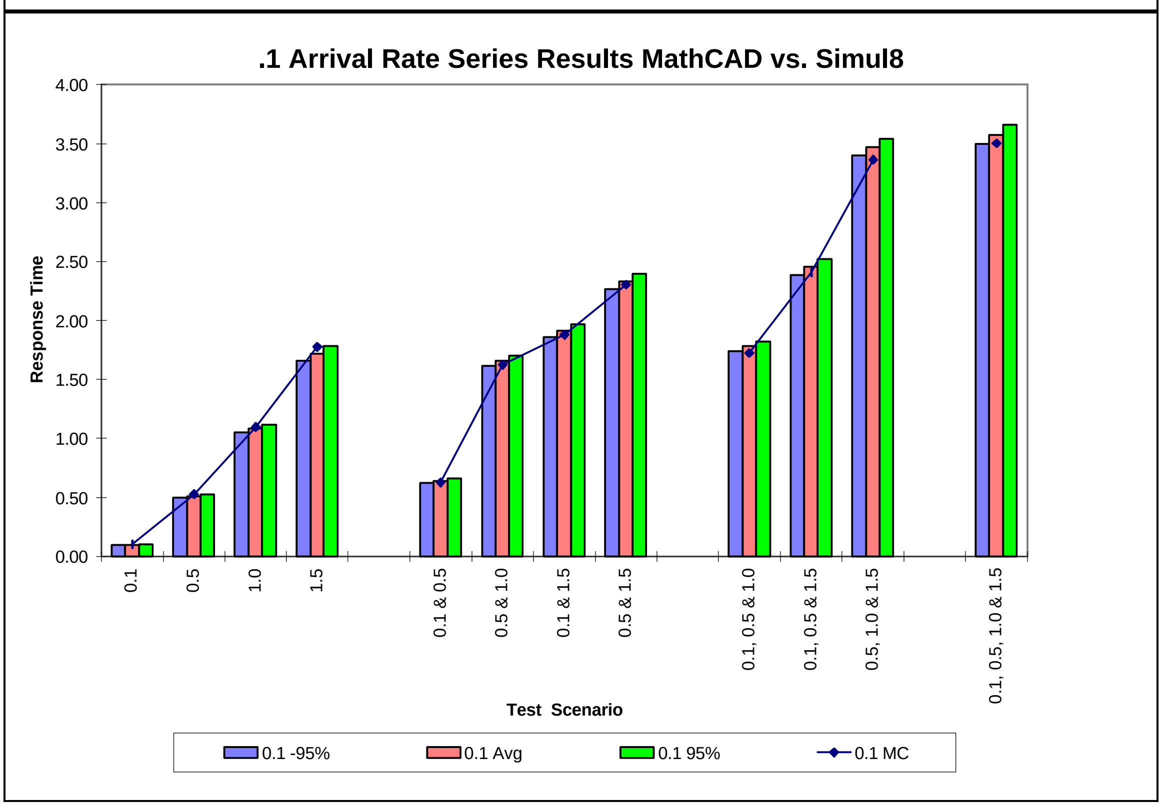
Table 12 High Service Time Scenarios

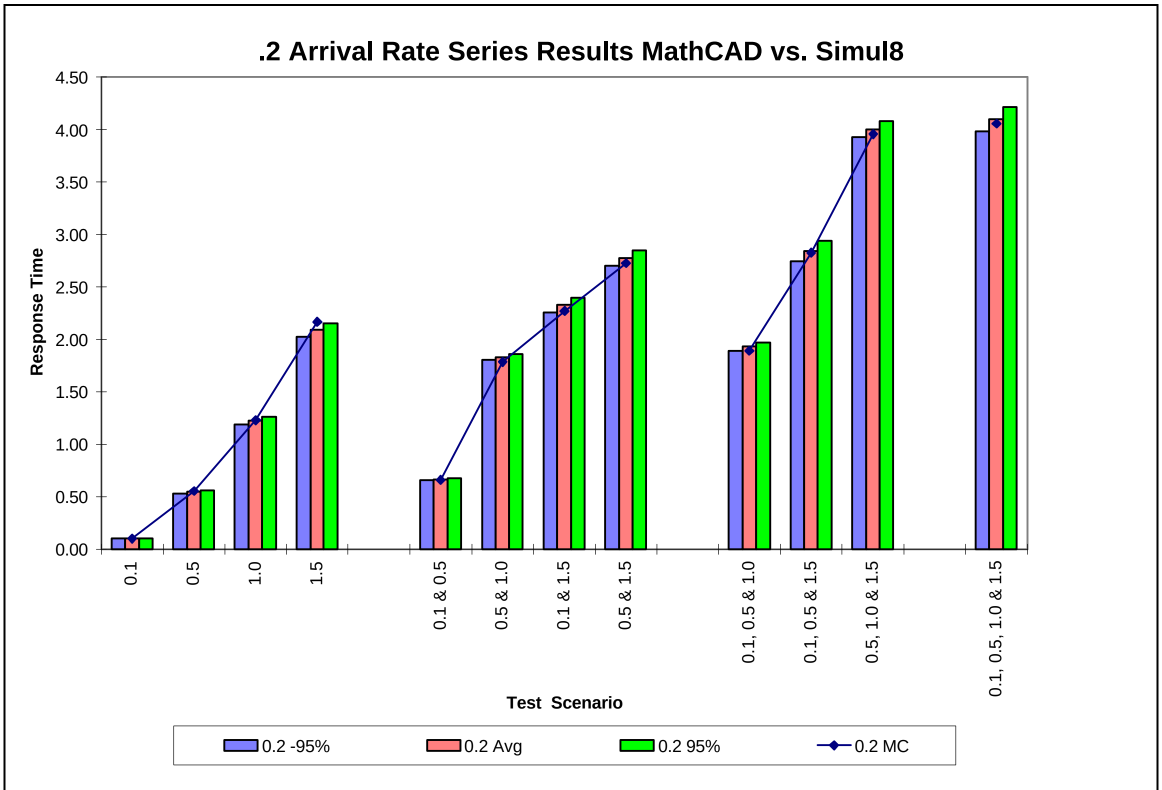
The last chart in this section is a similar comparison for the high service time servers shown in *Table 12 High Service Time Scenarios*, which also shows the service times for these scenarios. A check mark (✓) indicates which arrival rates were modeled for each service time. A blank indicates that the model saturated and did not produce usable results.

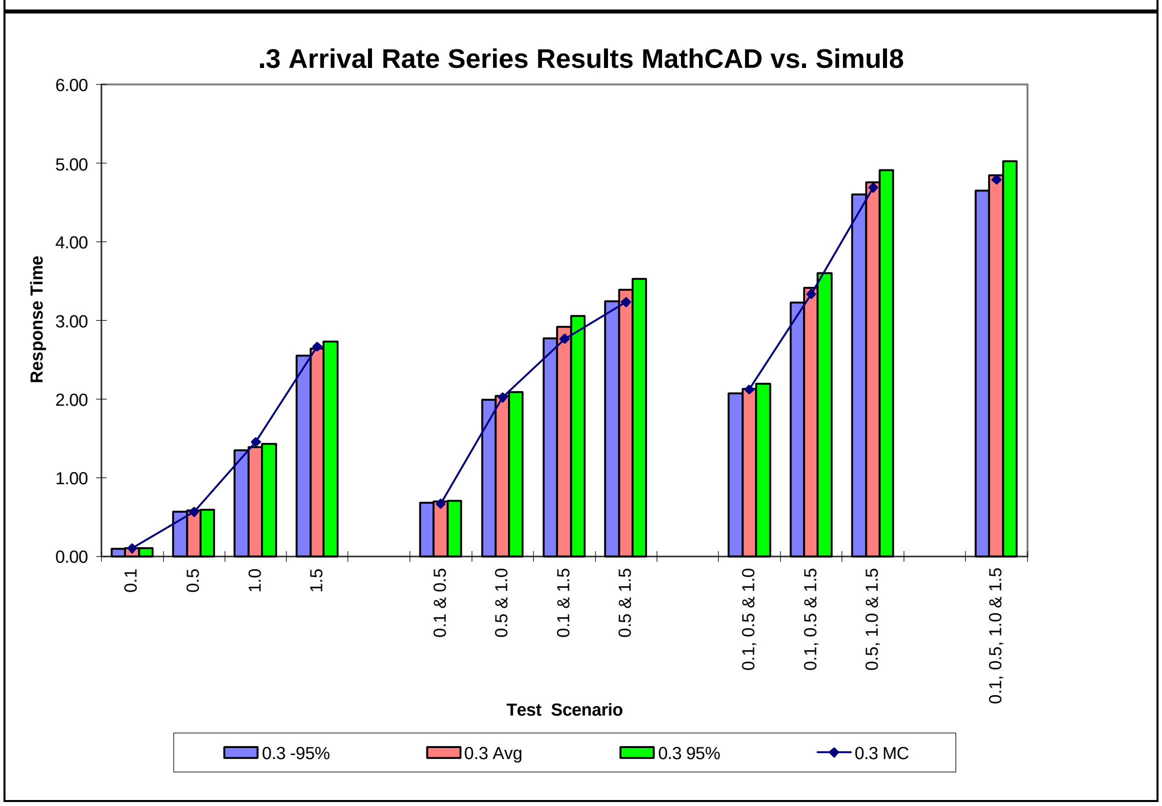
In most cases the MathCAD result is very close to the average response time and between the ±95% confidence limits. The few cases where the MathCAD results were significantly different were assumed to be caused by distribution variations in the MathCAD results because they were generated by single run trials. To determine if this was a reasonable assumption, a number of scenarios showing the difference were modeled with

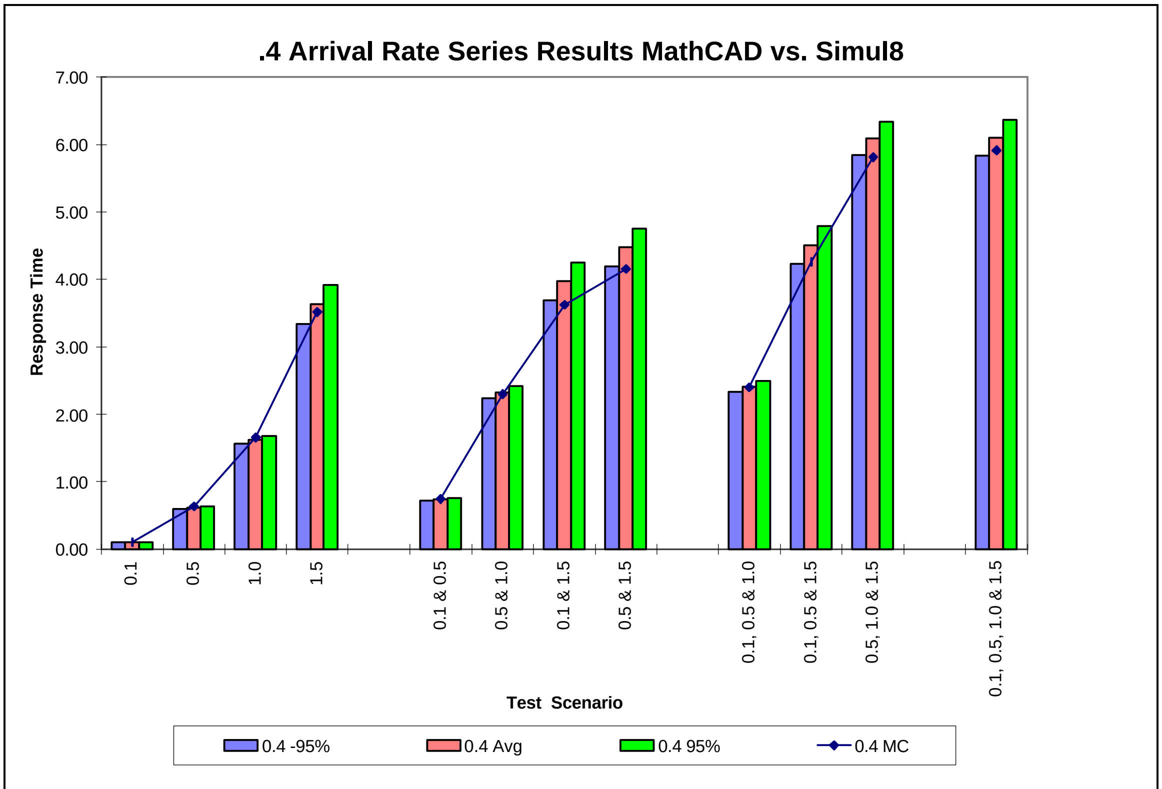
MathCAD using multiple run trials. These selected scenarios were: a single server with a service time of 1.5 and an arrival rate of 0.5, all of the scenarios for a single server with a service time of 25.0 and all of the scenarios for a single server with a service time of 75.0. The first was selected because most of the differences appeared to occur in a scenario that included this server. The last two were selected because they showed differences both significantly higher than and lower than the Simul8 results. Additional MathCAD model results were generated using seven model runs per trial, each with a different random number seed. These results were very consistent with the Simul8 model results and strongly indicate that the assumption was correct. Each of the multi-run MathCAD trials require significant effort and resources and it was determined that there was no additional benefit to conducting additional trials. These selected multiple run trials are indicated on the *.5 Arrival Rate Series Results* and the *High Service Time Results* charts as the series labeled *MC Trials Average*.

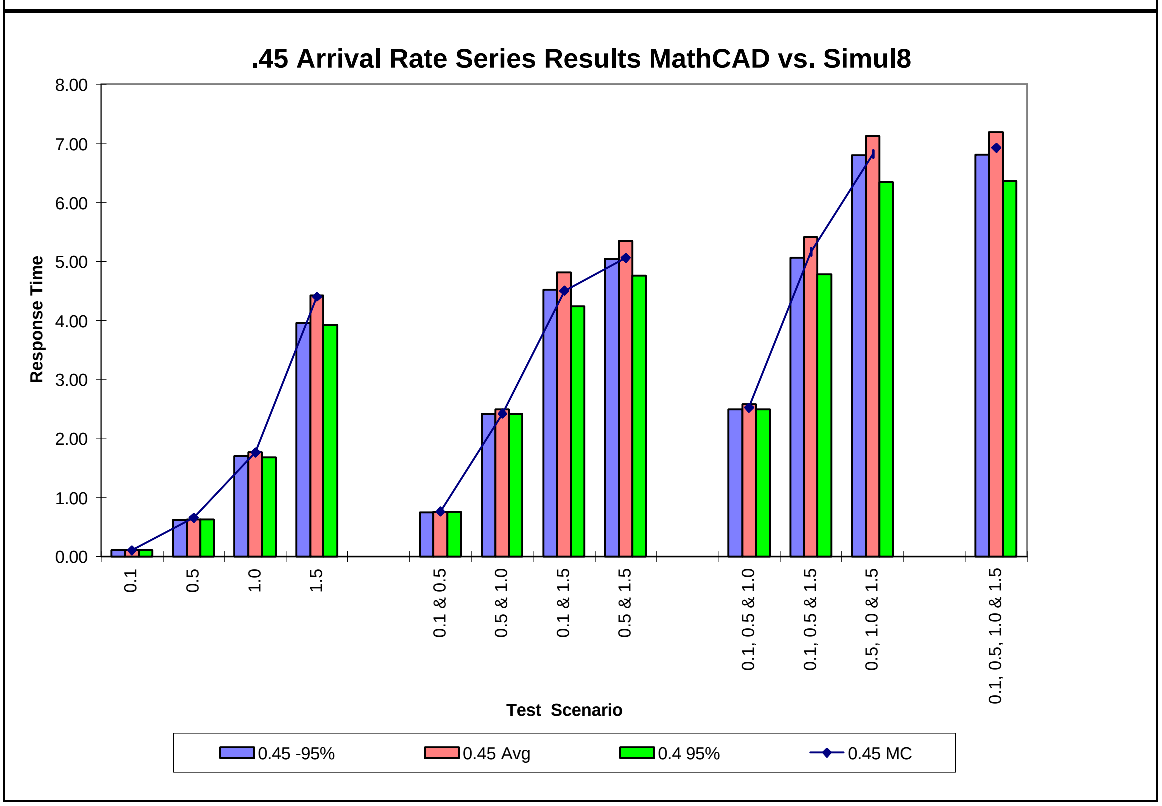

.01 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.01 -95%
0.01 Avg
0.01 95%
0.01 MC

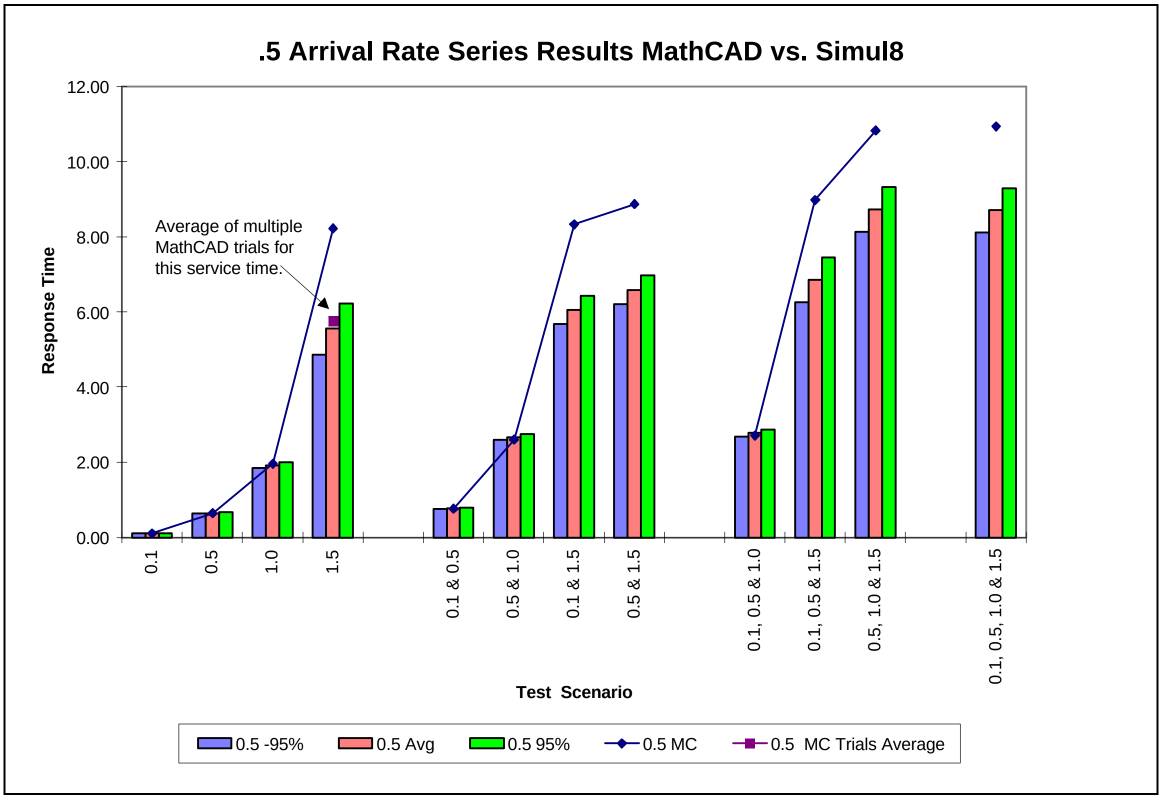

.1 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.1 -95%
0.1 Avg
0.1 95%
0.1 MC


.2 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.2 -95%
0.2 Avg
0.2 95%
0.2 MC

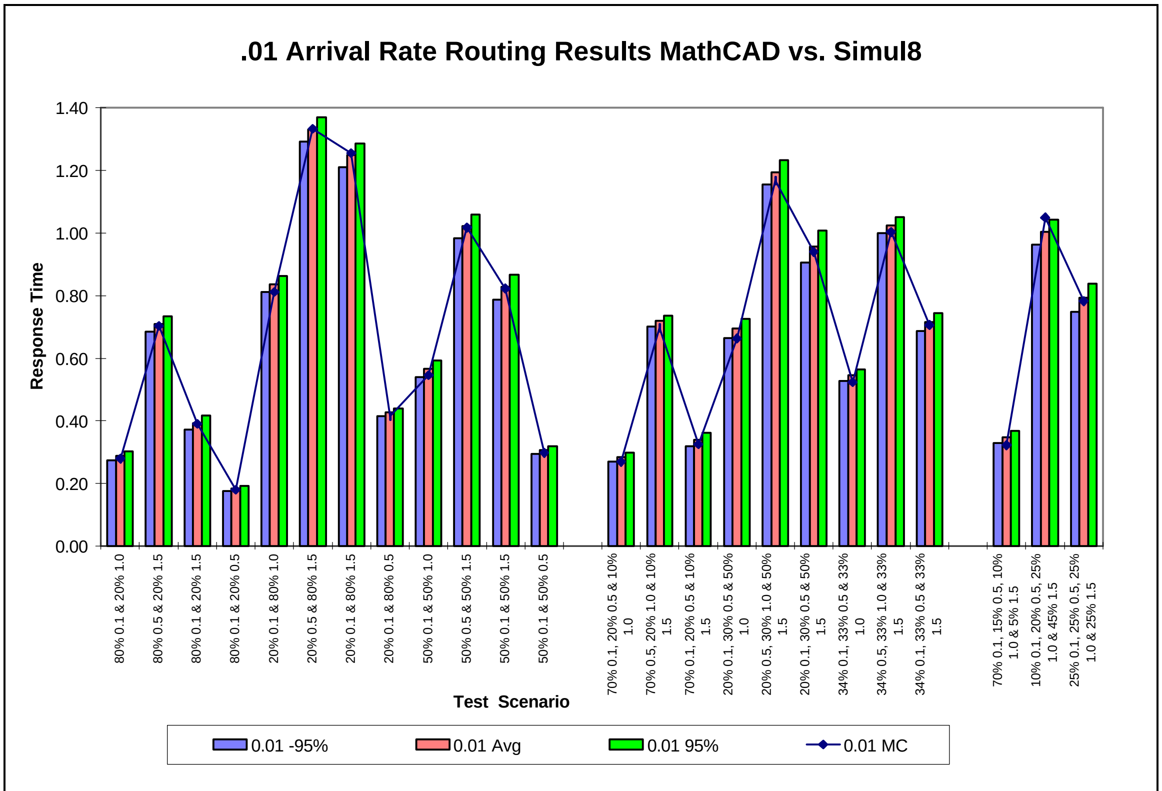

.3 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.3 -95%
0.3 Avg
0.3 95%
0.3 MC

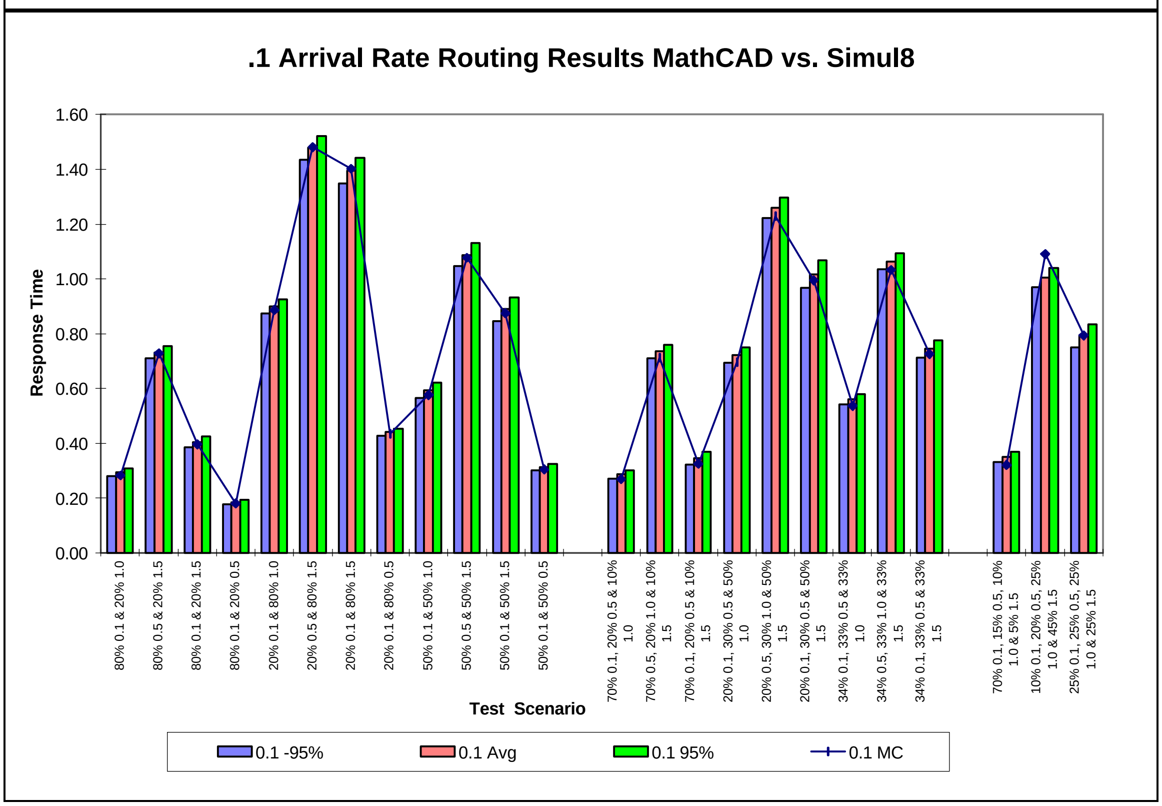

.4 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.4 -95%
0.4 Avg
0.4 95%
0.4 MC

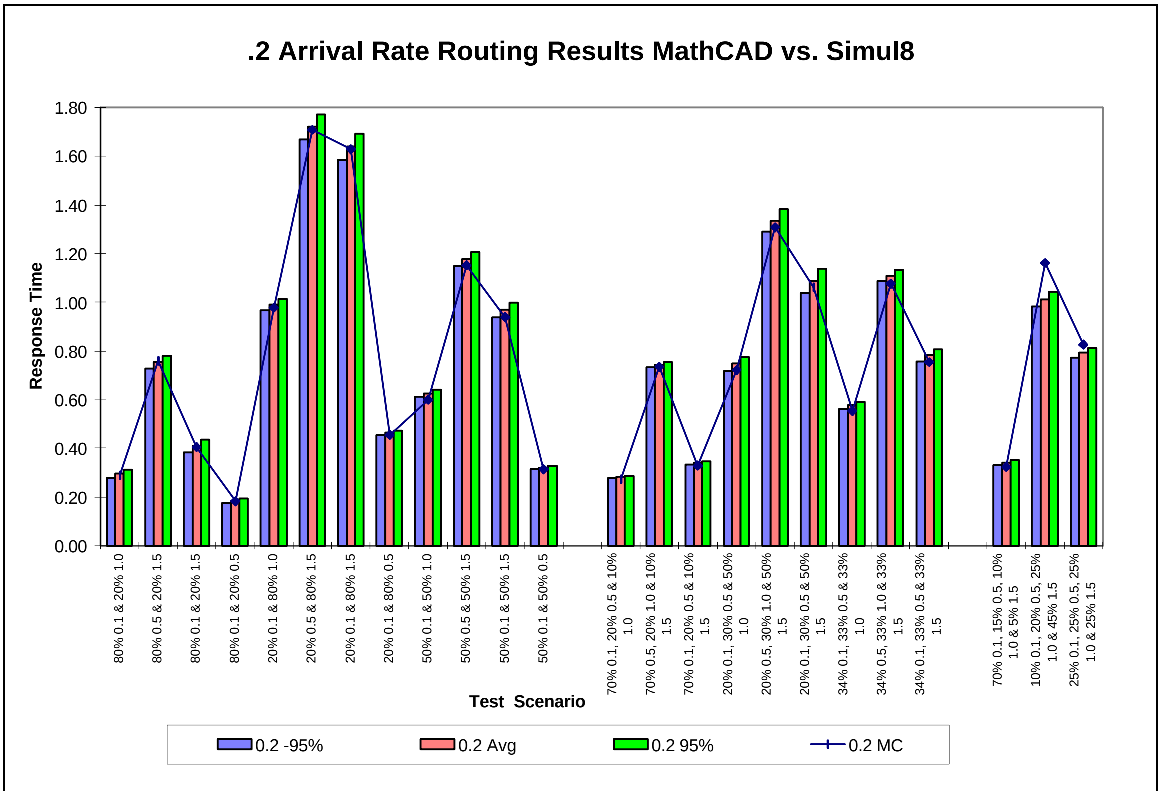

.45 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.45 -95%
0.45 Avg
0.4 95%
0.45 MC

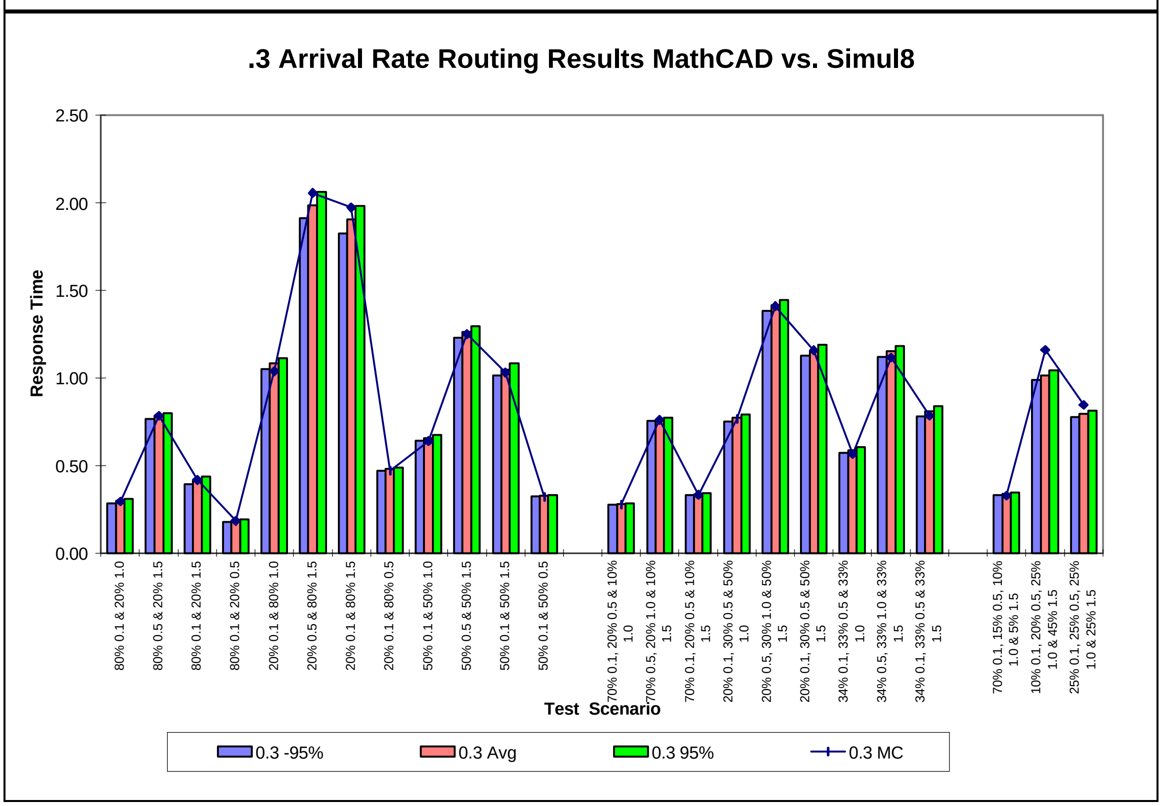


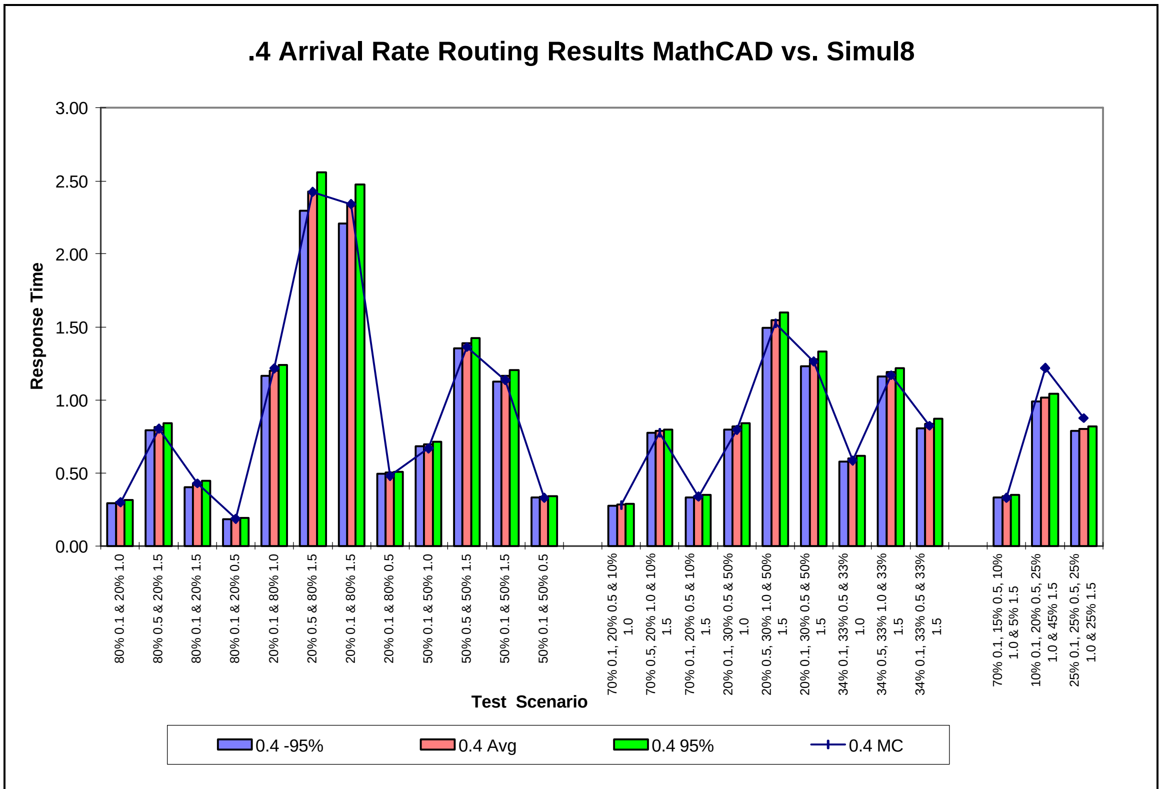
.5 Arrival Rate Series Results MathCAD vs. Simul8
Response Time
12.00
10.00
8.00
6.00
4.00
2.00
0.00
Average of multiple MathCAD trials for this service time.
0.1
0.5
1.0
1.5
0.1 & 0.5
0.5 & 1.0
0.1 & 1.5
0.5 & 1.5
0.1, 0.5 & 1.0
0.1, 0.5 & 1.5
0.5, 1.0 & 1.5
0.1, 0.5, 1.0 & 1.5
Test Scenario
0.5 -95%
0.5 Avg
0.5 95%
0.5 MC
0.5 MC Trials Average

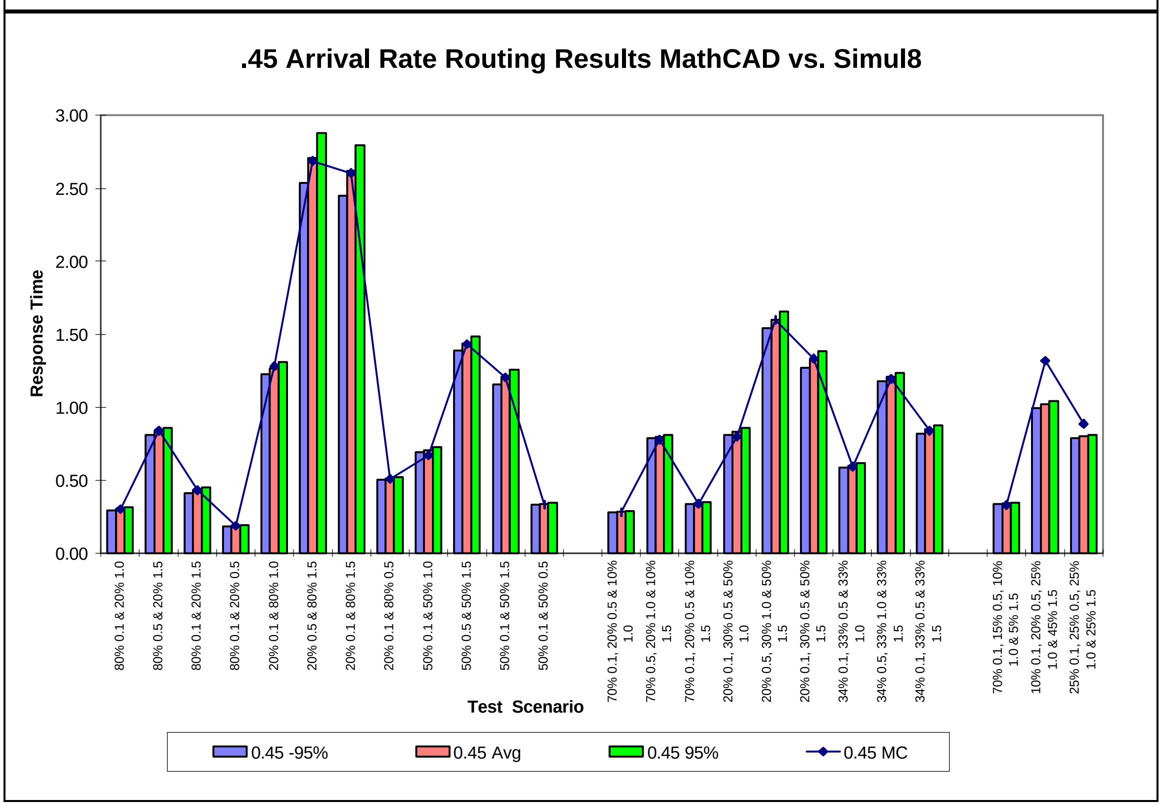

.01 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5
Test Scenario
0.01 -95%
0.01 Avg
0.01 95%
0.01 MC

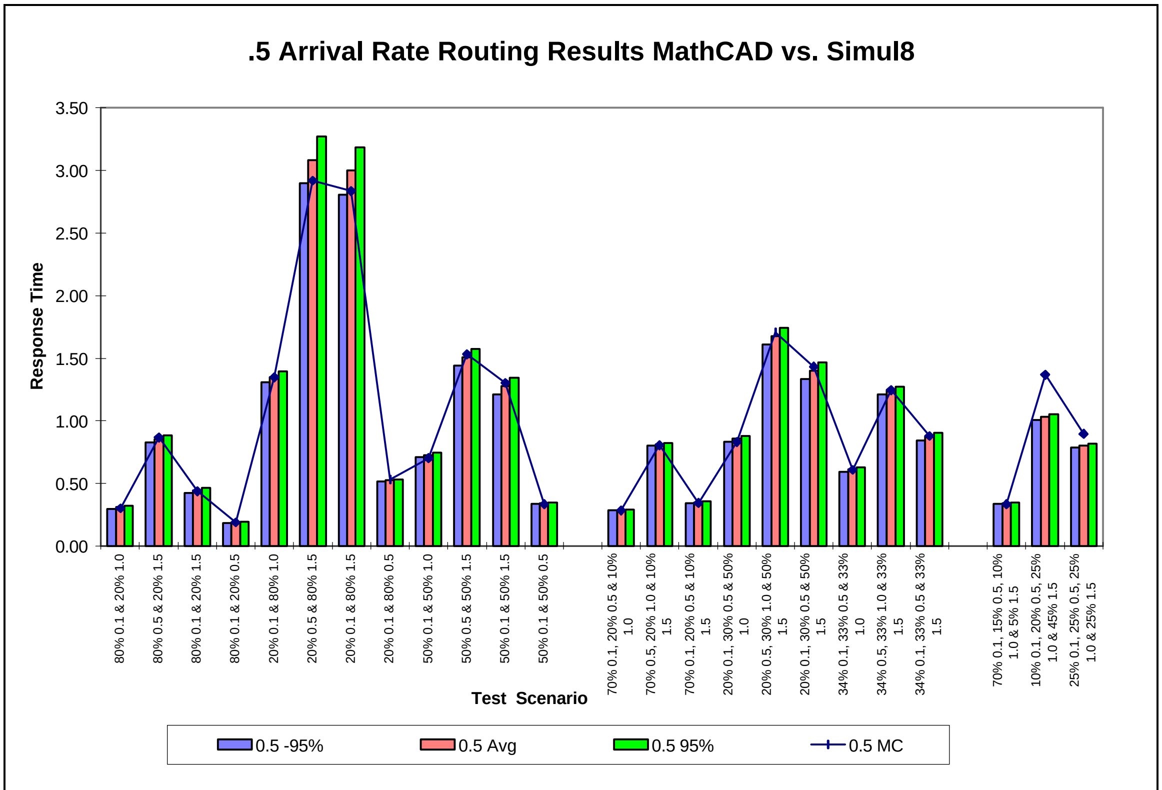

.1 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5
Test Scenario
0.1 -95%
0.1 Avg
0.1 95%
0.1 MC

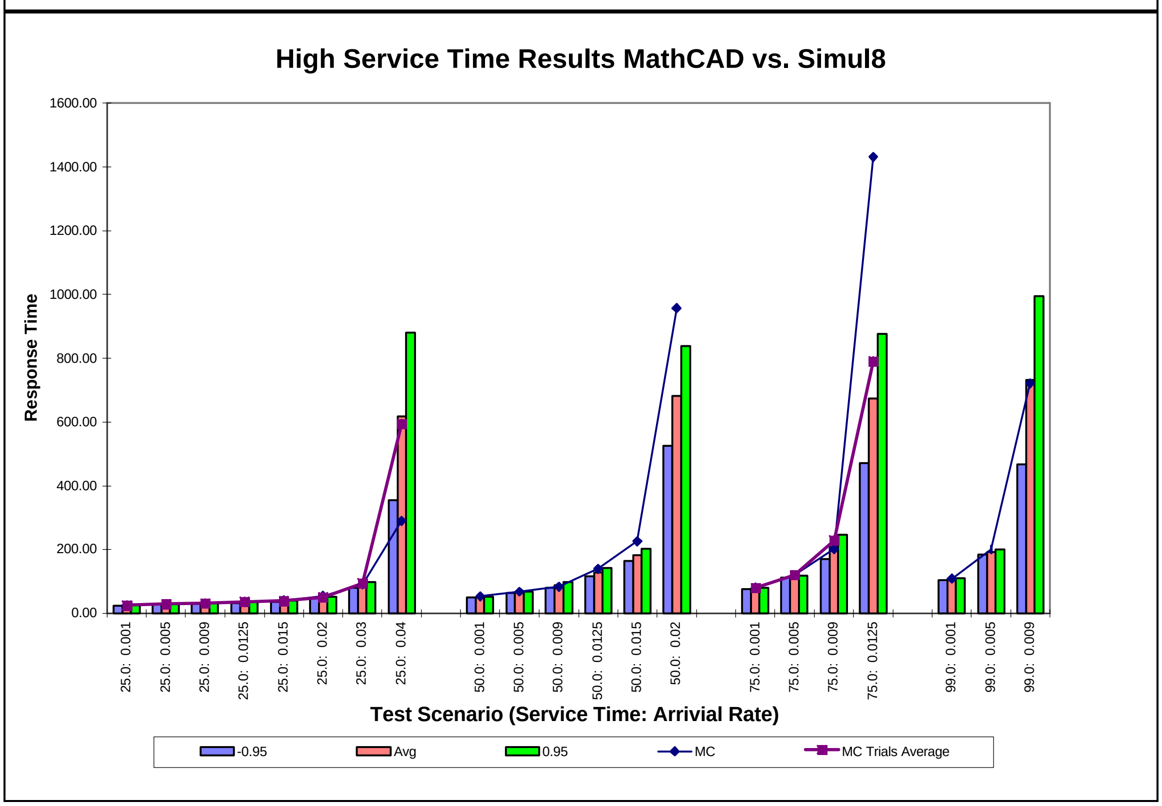

.2 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5
Test Scenario
0.2 -95%
0.2 Avg
0.2 95%
0.2 MC


.3 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
2.50
2.00
1.50
1.00
0.50
0.00
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5
Test Scenario
0.3 -95%
0.3 Avg
0.3 95%
0.3 MC


.4 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
Test Scenario
0.4 -95%
0.4 Avg
0.4 95%
0.4 MC
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5


.45 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
Test Scenario
0.45 -95%
0.45 Avg
0.45 95%
0.45 MC
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5


.5 Arrival Rate Routing Results MathCAD vs. Simul8
Response Time
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
80% 0.1 & 20% 1.0
80% 0.5 & 20% 1.5
80% 0.1 & 20% 1.5
80% 0.1 & 20% 0.5
20% 0.1 & 80% 1.0
20% 0.5 & 80% 1.5
20% 0.1 & 80% 1.5
20% 0.1 & 80% 0.5
50% 0.1 & 50% 1.0
50% 0.5 & 50% 1.5
50% 0.1 & 50% 1.5
50% 0.1 & 50% 0.5
70% 0.1, 20% 0.5 & 10% 1.0
70% 0.5, 20% 1.0 & 10% 1.5
70% 0.1, 20% 0.5 & 10% 1.5
20% 0.1, 30% 0.5 & 50% 1.0
20% 0.5, 30% 1.0 & 50% 1.5
20% 0.1, 30% 0.5 & 50% 1.5
34% 0.1, 33% 0.5 & 33% 1.0
34% 0.5, 33% 1.0 & 33% 1.5
34% 0.1, 33% 0.5 & 33% 1.5
70% 0.1, 15% 0.5, 10% 1.0 & 5% 1.5
10% 0.1, 20% 0.5, 25% 1.0 & 45% 1.5
25% 0.1, 25% 0.5, 25% 1.0 & 25% 1.5
Test Scenario
0.5 -95%
0.5 Avg
0.5 95%
0.5 MC


High Service Time Results MathCAD vs. Simul8
Response Time
1600.00
1400.00
1200.00
1000.00
800.00
600.00
400.00
200.00
0.00
25.0: 0.001
25.0: 0.005
25.0: 0.009
25.0: 0.0125
25.0: 0.015
25.0: 0.02
25.0: 0.03
25.0: 0.04
50.0: 0.001
50.0: 0.005
50.0: 0.009
50.0: 0.0125
50.0: 0.015
50.0: 0.02
75.0: 0.001
75.0: 0.005
75.0: 0.009
75.0: 0.0125
99.0: 0.001
99.0: 0.005
99.0: 0.009
Test Scenario (Service Time: Arrivial Rate)
-0.95
Avg
0.95
MC
MC Trials Average

7.7 Appendix G: Tool Baseline Comparison Results

This final set of comparisons establishes the validity of the Simul8 tool to provide baseline simulation results by comparing those results to both the MathCAD results (simulation and queuing theory) and the OpenQN results. These comparisons are shown on two charts.

Figure 61 Single Server Baseline Chart compares the Simul8 results for single server model test scenarios to all three sets of the other results for the same environment, MathCAD simulation, MathCAD queuing theory and OpenQN. This chart shows a very close correlation between all four of the groups of results. The **Test Scenario** names began with **M** (for model) and reflect the arrival rate and service time for each scenario in the

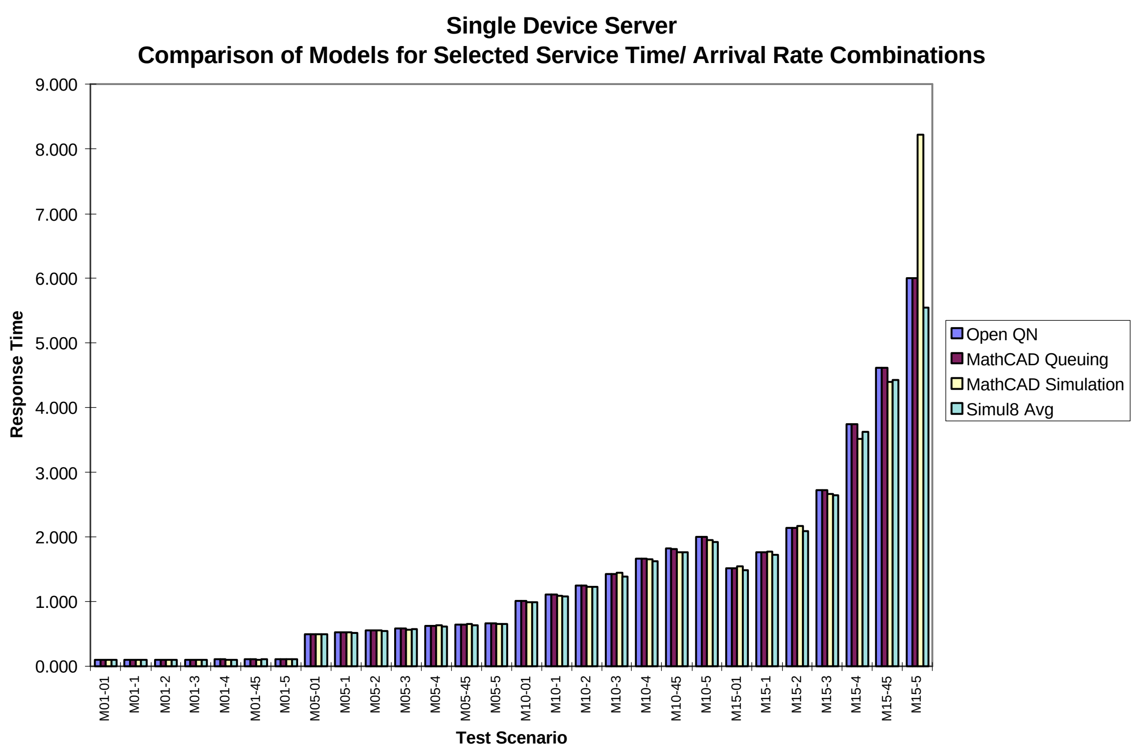


Figure 61 Single Server Baseline Chart

format **Mss-aa**, where **ss** is the service time with an assumed decimal between the digits (**01**=0.1) and **aa** is the arrival rate with an assumed decimal before the digits (**45**=.45). The large difference between the MathCAD simulation result and the other results for the far right test scenario (**M15-5**) was due to the single run trial for this model as discussed in section *7.5 Appendix E: MathCAD Formulae Results Charts* on page 180. The anomaly disappeared when the MathCAD model was rerun using a multi-run trial. That result is not reflected in this chart because it only shows the single run trial for consistency across the chart.

Figure 62 Multi-device Server Baseline Chart compares the Simul8 results for system (multi-device server) test scenarios to the same environment modeled with

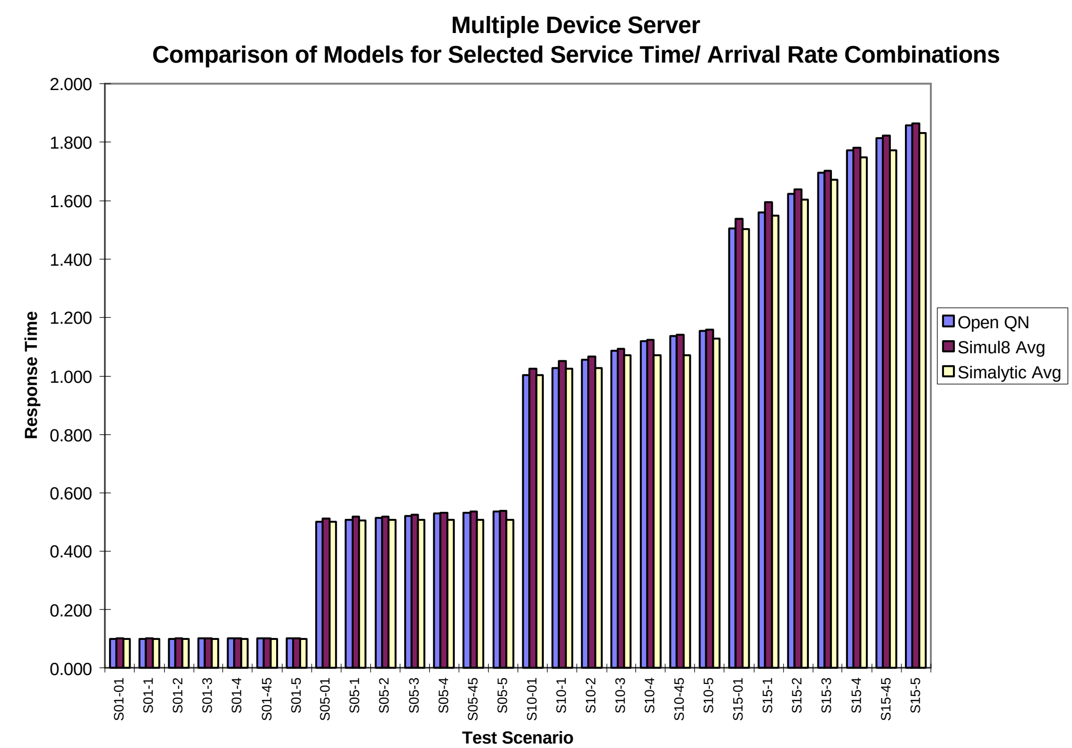


Figure 62 Multi-device Server Baseline Chart

OpenQN and with Simul8 using a Simalytic Function (these models were not implemented in MathCAD). The **Test Scenario** names began with **S** (for server) and reflect the arrival rate and service time for each scenario in the format **Sss-aa**, where **ss** is the service time with an assumed decimal between the digits (**01**=0.1) and **aa** is the arrival rate with an assumed decimal before the digits (**45**=.45). This chart shows the near identical results for all three of the modeling techniques, thus establishing the Simul8 modeling tools as valid to use for the baseline for comparisons with Simalytic Models and that Simalytic Models produce equivalent results for individual nodes.

7.8 Appendix H: Glossary

CICS	Customer Information Control System. TP system developed for IBM System/360 mainframe computer systems. Currently CICS is available from IBM for a number of operating systems (and the respective hardware platforms), including AIX, MVS, Windows NT and OS/2.
DB2	Data Base 2. A relational data base system developed for IBM System/360 mainframe computer systems. Currently DB2 is available from IBM for a number of operating systems (and the respective hardware platforms), including AIX, MVS, Windows NT and OS/2
GUI	Graphical User Interface. The interface between the user and a program that uses the computer's graphics capabilities to make the application easier to use, generally by representing program functions with icons the user clicks on with a mouse instead of typing application commands.
Informix	Informix is an object-relational database management system that is available on a large number of Unix systems from Informix Systems, Inc.
MPP	Massively Parallel Processing. Processors constructed of large numbers of CPU’s and connected using some type of communications paths (generally the CPU’s do not have access to the same memory and disks, i.e. “shared nothing”).

NFS	Network File System. Protocol used to allow one system to offer disk files to be shared by other systems over a network.
node	A computer system (including CPU, disk and network communications) used as a server in a client/server environment. In the context of a model, the term is used to mean a combination of server and queue or a submodel and queue that represent some relatively independent component of an application or provide service to an application.
OLTP	On-Line Transaction Processing. Generic term for developing an application design implemented with end-user terminals or workstations and a TP system.
responsiveness	The quality of something to respond to a request from a particular viewpoint. The responsiveness of a device can be its service time from the hardware viewpoint or the response time from the application viewpoint. The responsiveness of an application or transaction is the user's perception of response time. Responsiveness is good based on what is expected or required by that viewpoint.
SPE	Software Performance Engineering. The technique developed by Dr. Connie Smith as an early lifecycle method for designing and constructing software systems to meet the required performance objectives by identifying performance problems in the design phase (Smith 1990).
TP	Transaction Processor. A system component, generally considered an extension of the operating system, that provides a programming API, a

	user interface and resource management functions to schedule and execute transactions. TP systems generally provide services to control terminals or workstations so the transaction program can be developed independent of the required communications protocols and screen formatting commands.
TPC	Transaction Processing Performance Council. A series of benchmarks developed to provide objective comparisons between processors from different manufacturers. The individual benchmarks are designated by a suffix letter, such as TPC-A, TPC-B, TPC-C and TPC-D.
Tuxedo	A transactions processing system developed for Unix systems. Also referred to as middleware because it generally is implemented in the middle layer of a three tiered architecture. Currently Tuxedo is available from BEA Systems, Inc. for both Microsoft NT and a large number of Unix operating systems (and the respective hardware platforms). Client software is also available on MS-DOS, Macintosh OS , Windows 95 and OS/2.
transaction	A relatively small independent unit of work entered into the system by an end-user to receive some information as a response in near real-time. Transactions include entering an order at a terminal (business transaction), an SQL command (database transaction), some keystrokes followed by a carriage-return (interactive transaction)—whatever is meaningful from the end-user’s point-of-view.

	A database transaction is considered to be self-contained, or atomic, and is guaranteed to complete successfully or not at all. If an error prevents a partially-performed transaction from proceeding to completion, it must be "backed out'" to prevent the database being left in an inconsistent state.
trial	A series of model executions, or runs, used to generate a single result, usually averaged. Trials are used to reduce the influence of variations in arrival and service distributions in simulation models.

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